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This investigation was made in a secondary school where the learning of algebra was studied during three years (Grades 7-9). There were 119 pupils divided into two experimental groups each of which had a different course in algebra. The content of these courses was measured by the number of written exercises. Many ability tests and attitude ratings were presented to the pupils. Then, essential differences between pupils were extracted by using several multivariate methods. As a result, the following intervening variables for information processing were formed - Reasoning Ability, Numerical Ability, Attitude to Algebra, Simple Algebra were the most important variables when predicting school success in Algebra. Many differences between groups are stated in learning results. Differences between weak and bright pupils are presented. (RP)

Paavo Malinen

THE LEARNING OF ELEMENTARY ALGEBRA

An Empirical Investigation of the Results of
Learning in a Simplifield School Learning System

1969
Institute of Education
University of Helsinki

ABSTRACT

This investigation was made in a secondary school where the learning of algebra was followed during three years (Grades 7–9). The number of pupils was 119 and they were divided into two experimental groups which had different courses in Algebra. The content of these courses was measured by the number of written exercises. Many ability tests and attitude ratings were presented to the pupils. Then, essential differences between pupils were extracted by using several multivariate methods. As a result the following intervening variables for information processing were formed: Reasoning Ability, Numerical Ability, Attitude to Algebra, Simple Algebra, and Understanding. School success in Algebra was measured by 20 achievement tests and by the marks in Algebra. Attitude to Algebra and Simple Algebra were the most important variables when predicting school success in Algebra. Many differences between groups were stated in learning results. It was advantageous to use simple exercises which were divided into several teaching periods. Differences between weak and bright pupils were presented.

Kustannusosakeyhtiö Otavan kirjapaino, Helsinki 1969

PREFACE

This investigation was carried out during the years 1960–64 in Alppilan yhteislyseo, which is the state experimental secondary school of Finland. The many organizational arrangements for this investigation were made possible by the kind permission of the National Board of Schools and by the important support of Mr. Lars Frösén, who was at that time the headmaster of the school. I have received help also from many colleagues, especially from Mrs. Riitta Timonen, who organized her teaching in Algebra for three years according to the experimental design. I wish to thank my colleagues and also our pupils, who collaborated during the experimental period.

The planning of this investigation was made at the Institute of Education, University of Helsinki, under the supervision of Professor Matti Koskenniemi, who has given me much valuable advice concerning the theoretical foundation. Also the whole personnel of this institute has helped me in many ways, and two of them, Mr. Johannes Alikoski and Dr. Erkki A. Niskanen, have read the manuscript and given advice as to its final form. At the beginning and at the end of this investigation I had the opportunity of discussing with Dr. Ingvar Werdelin, Sweden, and obtained much advice, especially concerning the tests and experimentation. During the investigation many other research workers have helped me. I wish to thank all these friends for their invaluable support. I am indebted also to my assistants and the technical personnel of the computer centres for their share in this work. Mr. Anthony May has corrected the text and I am obliged to him for this aid.

For this investigation I have received financial support from Suomen Kulttuurirahasto and from the University of Helsinki. The Otava Publishing Company has helped me by agreeing to bear the costs of publication. I am grateful for all this support.

Jyväskylä, January 1969

Paavo Malinen

CONTENTS

Chapter 1. INTRODUCTION

Changes in mathematics teaching have been planned in many countries during this decade. The reasons for these changes in Finland are of many kinds. A comprehensive plan for mathematics teaching in the new compulsory school system (9 years) is needed. There is already a plan for changing the school system (Komitean mietintö, 1966:A 12), and a new law for its realization has recently been accepted (Suomen asetuskokoelma N:o 467, 1968). According to this, during the first six school years all pupils will study mathematics together. During the 7th to 9th school years there should be three "level groups" (Finnish: "tasoryhmä") in mathematics. This is grouping for instruction in a single subject, and the pupils themselves choose the level according to their wishes. The question of how this plan is to be realized is now urgent.

There have been strong trends among mathematicians to modernize mathematics teaching during the last decade (OEEC, 1961 a and b; Fehr, 1965; Markushevich, 1965). These trends have had their effect in Finland since the year 1960, when the Nordic Committee for the Modernization of School Mathematics began its work. This committee has already reported its results and has presented a new curriculum proposal for Grades 1–12 (Nordisk udredningsserie, 1967:9). A short report in English includes the aims and syllabus (Nordisk udredningsserie, 1967:11). For elementary education (Grades 1–9) there is only one basic course, which includes the main fields of numbers, geometry, measurement, probability, tables and diagrams, problem solving, and the nature of mathematics. The realization of this plan presupposes that the didactical implications which have been made, especially concerning the learning of modernized mathematics, will be taken into account. This didactical development has been brought about by psychologists of learning, experimental teachers and mathematicians (Magne, 1966; Unesco, 1966; etc.).

The reasons for the development of mathematics teaching are perhaps stronger in Finland than in many other countries, as the present results here are in some respects rather defective according to the International Study of Achievement in Mathematics (Husén, 1967). It was found that there were weak points in Finland, especially as to curricula, but generally it was not possible to draw any clear conclusions for improving the teaching of mathematics on the basis of this comparative study.

When reforming the present teaching of mathematics in Finland it is possible to make use of the considerable international experience of the modernized teaching of mathematics and also of the many studies concerning the effectiveness of mathematics teaching. However, all these do not solve the greatest problems concerning the organization of mathematics teaching. We list here the most important groups of problems, which should be solved before a reform is carried out.

1) Differences in aptitudes for mathematics learning have been investigated, but this has not been done in connection with the analysis of the modern syllabus. What kind of mathematics is best suited to all pupils? Is it useful to organize the content of the syllabus according to the spiral principle, i.e. dealing with the same principle on repeated occasions (c.f. Bruner, 1963; Nordisk udredningsserie, 1967:11, pp. 51–52)?

2) What individual differences are important for mathematics learning? What are the essential differences between a "slow" and a "bright" pupil? What is the didactical foundation for "level grouping" during Grades 7–9 (as in the new school system in Finland)?

3) When all pupils study in the same group, they will learn in different ways. Is teaching serving its purpose when in some situations it will give a general orientation only, where there are no great demands for skill? This idea has been presented, for example, in the report of the Nordic Committee (Nordisk udredningsserie, 1967:11, pp. 53–54). Is it possible to decrease training while increasing understanding?

4) How is it possible to lessen the difficulties in learning mathematics? Is some remedial

teaching needed and what is its role in the teaching system?

These problems are so extensive that they cannot be solved by a few isolated studies. These are not new problems, either, but they have received new importance because of the modernization plans. When I began this investigation in the year 1960, they formed the background in an undefined way. However, it was impossible to cope with them in this general form, because this presupposes a multitude of experiments concerning teaching procedures and special designs for teaching.

I was forced to restrict the problem by studying a specific learning situation in one school. Then, most teacher variables and the interactions between the pupils have been avoided. The object of the study is not the teaching of algebra, but the learning of algebra. Thus, I have concentrated here on some preliminary problems of learning in school situations. There are two essential areas to be investigated: (1) connections between intellectual variables, affective variables, and achievement variables of algebra, (2) the learning of different topics and different courses of algebra.

Much empirical research has been carried out concerning the learning of mathematics, especially during the last decade, but this does not give complete solutions to the problems of learning presented here. The theoretical background has been broadened during the time of this investigation and many books, which I refer to, have been printed during or after the empirical investigation. Some of the older books, whose results were preliminary, have therefore been omitted from the list of references, which includes mainly recent works, containing summaries and new data. I will not deal much with this theoretical foundation here, but concentrate on the empirical investigation.

Chapter 2. THE PLAN FOR THIS INVESTIGATION

Rationale of this Study

When I started planning this study in the spring of 1961, I had only limited knowledge of the modernizing trends and experiments in mathematics teaching (preliminary UICSM texts). That is why this investigation is not closely connected with the teaching of modern mathematics. However, the experimental course used in this study includes some modernized topics, which are now presented in a more developed form in the new programme of the Nordic Committee. Besides, I have tried to focus on those topics in the algebra course which are still found to contain skills necessary even in modernized courses (the simplification of expressions, the solving of equations and problems, etc).

The theoretical basis for the empirical investigations of didactics has been strongly criticised (Travers, 1962; Research Problems, 1960; Hendersson, 1963; Magne, 1966; etc.). At the beginning of this study I had difficulties in finding any firm foundation for it. The main idea was to organize during a long period such learning situations in school which could be controlled as well as possible. I had the comparatively rare opportunity of having the same pupils for five years and experimenting with them. Thus, it was possible to deal with problems which are connected with the development of performances during a long period. Information has been taken from a series of tests given, and in addition the author's own experiences of teaching have been used as a source. Two cross-sectional analyses of pupil performance and attitudes have been made. There is not information enough for a complete follow-up study, but enough for a development analysis.

In the future it may be possible to avail oneself of learning theories. This is an extensive problem, because many investigators have the optimistic view that the problems of didactics could be solved by using the theories of learning (Hill, 1964, pp. 51–52; Suppes, 1967, p. 5). However, it has been proved necessary to develop special theories of teaching for the description of school learning situations (Bruner, 1964; Gage, 1964; Koskenniemi, 1968; etc.). This experimental investigation will be concerned with learning, but I am mainly interested in touching on its implications for teaching. However, at present I cannot connect this investigation with the theories of learning or with the theories of teaching. That is why I have not presented any general hypothesis to be tested.

The first phase in the description of learning is to find suitable terms and variables. The terminology has not been precisely defined in this domain and I have been forced to use many terms which have been faulty described in previous reports. The tests have been planned to measure both cognitive and affective behaviour of pupils. The structure of our test variables is not well known and this leads to the structural analyses of all measured variables.

In the second phase test variables have been combined with behavioural variables using factor analytical methods. The reliability and constancy of these factor variables have also been studied. First, factors have been formed in a homogeneous test battery (e.g. attitude tests), but later the structure of variables in the factor analyses have been more heterogeneous. Thus, there have been different aims in these factor analyses. The interpretation of factors is defective because there are many factors which are not of interest to further investigation. The analysis of the more interesting factors has been made later in the total analysis.

This analysis leads to the structuring of our variables. It is also necessary to construct suitable variables for the description of school learning. The lack of a theoretical basis results in our being unable to estimate beforehand how far we can proceed in this direction.

The third phase is to analyse achievement in algebra and evaluate its relationship with the teaching. This can be done separately, but we have used here the structural analysis as a back-

ground. This investigation of levels of achievements is closely connected with the didactics of mathematics, but our experimental design is not intended for the precise planning of a curriculum.

From the beginning we have omitted any plan for investigating the teaching process. The only information about factors external to the learning process is the number of written exercises. Thus, using the terminology of information theory, we have a one-pupil information processing system. This is an open system, but the external inputs are selected from a small domain.

We have also omitted a comparison of results of boys and girls. This is not relevant to our aims as presented in Chapter 1. There are differences of development between boys and girls in these grades, but this probably does not prevent their being combined here. Werdelin has compared the results of boys and girls and used the test material like the one employed in the present study. The most essential differences are to be seen in the structure of space factors (Werdelin, 1960, pp. 84–85; Werdelin, 1968 a, p. 128).

During the years when this study was carried out, the theories of learning developed and the planning of experimental investigations grew further. However, the design in the well-planned Long Term Study of the School Mathematics Study Group (SMSG, 1962; Romberg & Wilson, 1968) is still rather similar to the one of my study. There has been improvement in the control of learning results and in the organization of experimental design (Werdelin, 1965; NCTM, 1967; Ekman, 1968; etc.), but most studies are still comparisons of methods used during short periods of teaching. They cannot yet form a basis for the planning of a curriculum.

In this study I have not used difference scores when investigating the development from Grade 7 to Grade 9. After the analysis of our variables it was obvious that equal difference scores did not have the same precise meaning at different points of the scale. We will assume that the variables have been measured using the interval scale, but there is a little unevenness in scales and this accumulates easily in difference scores. Because the constancy coefficients were rather high, the difference scores would be small and the stochastic variance great. Thus, the difficulties which are common when using difference scores (Bereiter, 1963) are present here.

Educational Theories of Mathematical Thinking

There are different kinds of theories concerning the learning of mathematics. We can give here only an outline of the way in which these have been used as a basis for this research.

In special situations conclusions have been drawn from general theories of learning but most of them do not take individual differences into account and this diminishes their applicability in our design (Du Bois, 1962, p. 66; Glaser, 1967, p. 14). A summary of the investigations concerning individual differences in learning appears in a report of a symposium (Gagné, 1967 b). For our purposes the article by Anderson (1967) is the most important one. This has as its scope the comparison of aptitude factors and performances, which is the purpose also of our investigation. However, this article does not provide a theoretical framework which can be used for the present research.

The theoretical foundation of school learning situations has been obtained from the general presentations of instruction and teaching. Piaget's developmental psychology has affected the formulation of theories of concept learning and principle learning, and the learning of mathematics has been a good area of application. A well-known system based on Piaget's ideas was constructed by Dienes. He has presented a skeleton theory which he calls a theory of mathematics learning (Dienes, 1960, pp. 31–48). Later on he has presented a number of other books (Dienes, 1964; Dienes & Jeeves, 1965; etc.), but they consist mainly of theoretical considerations of the instructional system for teaching. The same outlines as presented by Dienes are also to be seen in many other investigations concerning mathematics education (Research Problems, 1960; Hendersson, 1963;

Magne, 1966; Unesco, 1966; NCTM, 1967). The discovery principle and the process of instruction are common topics for discussion. The concept "understanding" has been given a central position in the description of the learning process, and it needs to be analysed more precisely.

The theory that there are different levels of understanding in mathematics has been presented already in The Learning of Mathematics (NCTM, 1953, pp. 8–10). These levels are end products of learning and they can be seen in implicit or explicit form in any of the descriptions of mathematics learning. This concept has also been interpreted as different forms of thinking. In an analysis by Gagné (1965, pp. 175–180) these have been listed as sequences. We do not discuss here whether Gagné's levels present different learning processes, because we cannot analyse the learning processes. We need his levels only to describe the level of understanding.

There are also levels in the taxonomy of the cognitive domain aims (Bloom, 1956), which can be reached by using different ways of thinking. The construction of this taxonomy may be on the same basis as the construction of levels of understanding.

We can describe the concept "levels of understanding" when the pupils work with the same task by using chaining, verbal sequences, multiple discriminations, concepts or principles (c.f. Gagné, 1965, pp. 175–180). The solving of a simple equation may be possible in all these cases. The solving of more complex problems assumes a higher level of understanding. It is not obvious that we could measure a variable which we could call understanding in this meaning, i.e. which would indicate the level for the processing in thinking (Williams, 1965, pp. 38–39). We will discuss this problem later (see page 49).

An important theoretical basis for this study was, at least at the beginning, the automatization theory of Werdelin (1958, pp. 172–179). He completed this later (Malinen, 1961, p. 27) and it was then verified in some simple situations. According to this, there will be a change in the factorial content of tests from a more "general" structure to a more specific location. There are corresponding results of changes in factor structures also in other studies (Heinonen, 1964, p. 182), but only in the psychomotor domain has there been a more systematic treatment (Fleishman, 1967).

We can present much evidence that there are changes in factor structure during the training process, but it is also possible that there are differences in the factor structure between pupils. A "dull" pupil has perhaps a different factor structure from a "bright" pupil. This leads to new problems concerning task analysis. What kind of skills are important for slow achievers and for bright achievers? If there are differences, there should be a connection with the levels of understanding. If so, we cannot describe didactics as a system where there is a linear relationship between a method and its results. If a certain method is suitable for a bright pupil, it may be unfavourable for a dull pupil. These problems have not been discussed much, but outlines are to be found in Anderson (1967), Cronbach (1967), and Zeaman & House (1967).

To sum up, the theoretical considerations concerning mathematics education are incoherent for precise hypothesis formulation. Our experimental design assumes the consideration of individual differences in many dimensions and for this purpose we can get only some outlines.

A Simplified Model for Mathematics Learning

For our experimental design we need a model for the investigation of the learning process of a pupil. One suitable system for this purpose is presented by Ryans (1963; 1965). For this study we must simplify his model in many ways and we can use only its outer frames, because we do not study learning processes in detail.

We must reduce the external information inputs to include only information concerning algebra. It would have been possible to treat the teacher information processing, which gives as its result the teacher information output (teaching during the lessons, exercises etc.). This is the same as the pupil's external input.

We have not presented a system for teacher information, however, because we have measured only the amount of written exercises. Then, the only differences between experimental groups in external inputs are, according to our assumptions, the differences between the courses. The individual differences in external input receiving have been omitted.

We will not investigate pupil information processing. This would be a very interesting object for the study of understanding, because it is connected with processing. However, we have measured only information outputs while using our tests.

We must reduce the internal information inputs to include those capabilities which have been measured. These areas have been described as characteristic abilities in the cognitive domain, characteristic attitudes in the affective domain, and characteristic achievements in algebra as the information base. The pupils differ in these characteristics and we can form variables to describe these differences.

This simplified model for pupil information procedure is presented in Figure 2.1. We have written there only those names which have been used in our system.

In the experimental design we can distinguish three groups of variables:

1) Independent variables: differences in the amount and quality of the external information.

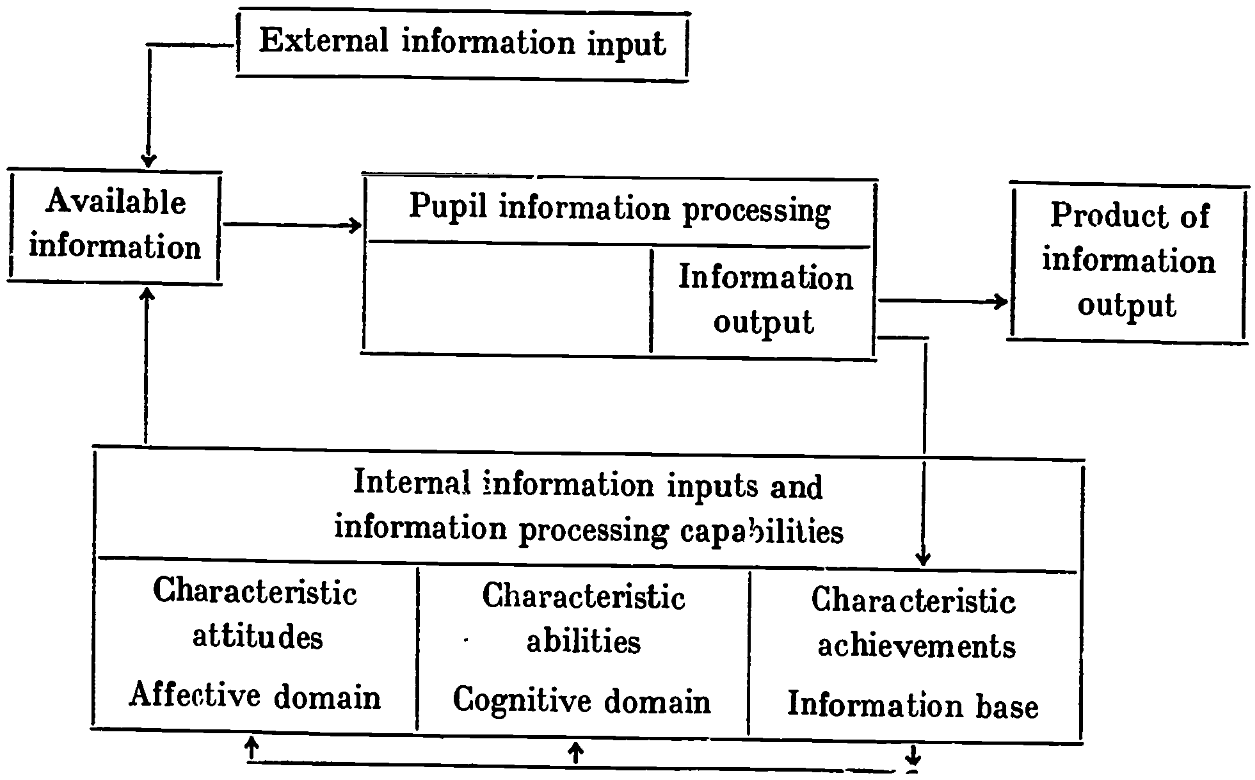
2) Intervening variables: differences between pupils in information processing. This group includes attitudes, abilities, previous achievements and other means for the learning process.

3) Dependent variables: the results of the learning process (new achievements and new forms of affective behaviour).

We cannot distinguish these groups precisely, because some achievement or attitude variables can in one situation be a result of learning but later on in another situation an intervening variable. When for instance a new topic of algebra has been taught and afterwards tested, we get from this testing a dependent variable. Later on, the knowledge in this domain of algebra can affect the performance in another topic. If we then test previous knowledge, we have it as an intervening variable. We have not distinguished intervening and dependent variables in this model, but we make this distinction in Chapter 5. So far we are speaking only about information output variables.

There are also difficulties in differentiating between ability variables and achievement variables. We make use of those ability factors, which have been reliably measured in many factor analyses (Ahmavaara, 1957). The abili-

Figure 2.1. A simplified model for pupil information procedure



ties are concerned with the crystallized intelligence as analysed by J. L. Horn (Pawlik, 1966, pp. 553–561). If there are changes in the processing of the test, it will give us an achievement variable.

This model does not include a theory of learning. It gives only a formal system for the description of our experimental situation in a revised form. Using this model we can clearly present the boundaries for the future treatment.

Task Analysis: Aims and Tasks in Algebra Teaching

Modern aims in mathematics teaching are presented in the report of the Nordic Committee for the Modernization of School Mathematics (Nordisk udredningsserie, 1967:11, pp. 8–47). There are 1) cognitive domain aims, such as understanding the basic concepts in the mathematics curriculum, 2) affective domain aims, such as experiencing mathematics as a living subject and experiencing pleasure through work on the subject (affective aspects of attitude), or giving an insight into the aesthetic values of mathematics (cognitive aspects of attitude), and 3) mean aims, such as helping students master new mathematical topics through independent mathematical study.

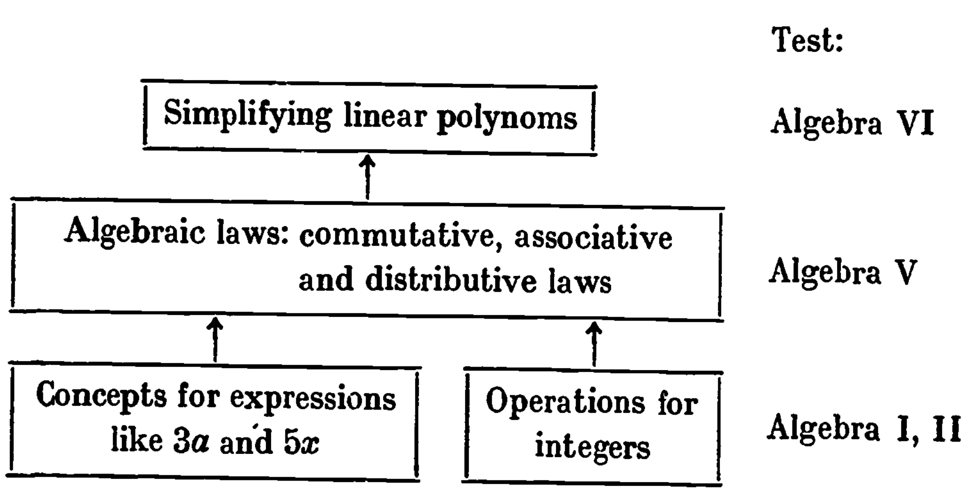
It is possible to form a hierarchical system of aims in the cognitive domain similar to Bloom's taxonomy (1956) which forms sequences in the dimension of the intrinsic of knowledge. For the learning of algebra this means that there are different levels of understanding from the superficial knowledge of names to the understanding of principles and to the analysis of proofs in the algebraic systems. There are also levels in the aims and the lower level must be reached before the higher level.

This taxonomy can be held as a base when describing the aims in the cognitive domain as in the National Longitudinal Study of Mathematical Abilities (Romberg & Wilson, 1968). In Chapter 3 we will analyse the content of our tests using this as a basis. We take here only one detail.

There are always given prerequisites for the learning of a new topic in mathematics. Thus, several topics can be formed as a learning set (Wallen & Travers, 1963; Gagné, 1967 a; Kersh, 1967). Such analyses of topics are needed for a well-planned learning system. For our material, a simple system can be formed concerning the simplifying of algebraic expressions like $2a + 3b + 5a$. The structure of topics for this task is in Figure 2.2. There are three tests, 'Algebra I', 'Algebra II' and 'Algebra V', which measure the mediating processes before the complex task will be measured by the test 'Algebra VI'.

It may be that the pupils solve the tasks in

Figure 2.2. Learning structure for simplifying linear polynoms



the test 'Algebra VI' using different mental processes. Some can have a good understanding of the laws of algebra and they solve these problems on the level of the learning of principles. Some have learned these laws as verbal sentences and use them as verbal advice like: "You can combine only such terms where the letters are equal". While knowing the results in the tests 'Algebra I, II' and 'Algebra V', we cannot estimate with certainty this process or the result of the test 'Algebra VI'. We can only estimate if good performances in the mediating processes are important for the result in this criterion test. There is still a great difference between the analysis of a learning set and the corresponding process of learning. The mediating processes and the mediating aims must first be analysed, and in our situation, a precise analysis is not possible.

The cognitive aims in the teaching of algebra will also be described here using the topics of the syllabus and the levels of understanding. The forming of aims changes during the learning process. When learning simple topics of the syllabus, the aim is to obtain a good understanding of the principles in this process and a good performance of these tasks. We cannot see beforehand if a high level of understanding is needed to get a correct performance. There is a strong trend in the didactics of mathematics to increase the level of understanding and there are discovery methods for reaching this aim. This means working with principles which grow from concrete problem situations. A review of these trends can be found in Hendersson's work (1963) and an extensive description in monographs written by Polya (1962; 1965). We postulate here that the aim is to understand principles, but we do not claim a constant level of understanding. Ability to obtain correct answers is another aim and this will be evaluated independently of the other aims.

When transferring to the more advanced topics of algebra we must change our aims. The learning of earlier topics is now only a means for attaining this aim, if we have subordinate topics. We also have such means which are common to all learning processes, e.g. capabilities. We can take some means as the aims, such as the ability to work independently. Then, it would be possible to take as one aim the common ability to understand mathematics. We have not, however, measured such a variable and it would perhaps be on the level of synthesis in Bloom's taxonomy. Thus, the learning of the last topics of the course is the last measured aim. These tests include application and are not very distinct from other tests as to the level in the taxonomy.

The aims of the study of the affective domain have been presented in Krathwohl's hierarchical system (1964). It is difficult to group our tests according to these levels, because on the whole they measure some kind of valuing of algebra. The 'Aspiration Level Test' (see Chapter 4) perhaps measures the higher level, the organization of a value system, but in a concise and superficial meaning. In any case, we can call all of them attitude tests. Thus, using this classification it is difficult to point out which tests would measure the means for learning processes and which would measure the final aims in the affective domain.

There is also another classification of the affective domain behaviour concerning attitudes: the cognitive component (beliefs or opinions) and the affective component (affective attitudes) (Fishbein, 1966; Karvonen, 1967). There are different opinions as to whether beliefs cause affective attitudes or *vice versa*. In our situation we suppose that there is first an affective attitude towards the learning of algebra and then beliefs grow about the necessity of learning algebra. Then, the affective attitude is a means for later learning results in this domain. We can accept that the reality in the aspiration level is a measure of the cognitive component, and also in this meaning it is the best measured variable for the aim.

This analysis of the aims in the teaching of mathematics is very defective, but we have concentrated on the aims which have been measured in this investigation. We have omitted such aims as creativity, ability to work independently, etc. Most of them are common aims which can be presented independently of the subject. We have measured common school success, but we do not regard it as a variable which measures aims, because it is not our purpose to analyse common success in school.

After the definition of aims we must analyse the tasks for reaching these aims. Tasks can be

described in terms of loadings on reference variables according to Anderson (1967, p. 67). We can criticise this definition, because here the tasks are dependent on the total reference system in which the school success will be described. When forming patterns of loadings we get a description of the relative importance of different intervening variables.

In our situation the tasks concern previous knowledge of algebra, willingness to learn algebra, and ability to learn algebra. The same performance can be reached using different profiles of task variables. Low ability can partly be compensated with high motivation etc. Thus, it is difficult to present minimum tasks which must be fulfilled to reach a constant aim. There may be individual differences in the profiles of task variables. It may be necessary to form different aims for different pupils to get optimal learning results. Some pupils need less training than others, but more discussion of principles etc. Those differences are accepted in practical school work, but there are only few hints about that in task analysis, such as Duncan's short article (1967).

In our investigation we can think of different profiles and different learning processes for good achievers and for weak achievers, but we have already stated that the separation of the processes is almost impossible. The possibilities would increase, if we could further vary the independent variables. Thus, we cannot proceed far in the task analysis, but these preliminary steps are also important, because this has not been an important object for empirical investigation.

The Experimental Design

Subjects

The empirical investigation of this report was carried out in Alppilan yhteislyseo, which is the state experimental school of Finland. It is a secondary school which receives its pupils after four school years in elementary school. Schooling in Finland begins at the age of seven years and so pupils enter secondary school at the age of about eleven years. Some are one year older because they have been five years in elementary school.

This school is located in Helsinki and at the beginning of this investigation more than one half of the total age group in the town entered secondary schools. There is no objective data about the level of performance of the pupils of this school as compared to the whole age group, but we can suppose that they form a rather typical sample from the upper fifty per cent of urban pupils. The selection of the pupils for secondary schooling was not based on a very effective entrance examination and some pupils with a lower performance level also entered this school.

This investigation was carried out with the pupils who entered this school in the autumn of 1959 at the lowest grade, which we call Grade 5. The education in mathematics was then followed for five years, i.e. from Grade 5 to Grade 9. These grades form the lower secondary school (Finnish: "keskikoulu") in Finland. During this time only a few students left the school and a few had to repeat the grade.

All pupils in the lower secondary school have the same syllabus in mathematics and they have 4 mathematics lessons (5 in Grade 5), each of 45 minutes' duration, per week. All the time they must take more than ten subjects, including two foreign languages. Some of these pupils studied English and Swedish, the others German and Swedish.

The experimental group consisted at the beginning of 143 pupils, and they were divided into four parallel sections which we call classes A, B, C, and D. These were formed according to the pupils' choices of foreign language without any thought of a streaming system or level

Table 2.1. Pupils Taking Part in the Experiment

	Class								
	A		B		C		D		
	b	g	b	g	b	g	b	g	Total
Entrants to Grade 5	9	26	17	19	17	18	12	25	143
Transfers to other schools	1	1	—	1	2	2	—	—	7
Repeaters	1	1*	5	1	1	2	3	3	17
Experimental group in Grade 9	7	24	12	17	14	14	9	22	119

* Because of a long illness.

grouping. During the time of this investigation there were no apparent differences in the levels of performance of these classes. Pupils in the same class studied for about 30 weekly hours together and during these five years they formed rather firm social groups.

Transfers from this school to other schools were rare (7 pupils) among these pupils during that time, and it has not caused any essential change in the structure of this group. The number of pupils who repeated Grades 6, 7 or 8 was somewhat higher, namely 17. They did not take part in the final study in Grade 9, and this causes a problem of selection which we will investigate further in Chapter 6. The numbers of pupils and the changes in the experimental group are presented in Table 2.1.

There are differences between the structures of the classes, because the number of boys is less in classes A and D. We do not compare the classes separately, but combine classes A and B, and correspondingly C and D. The proportion of boys is well balanced in these groups.

During these years some new pupils entered these classes, but they were not included in the experimental group. During the final study the average age of the pupils was 15.8 years and the standard deviation 0.6 years.

The Teaching of Algebra

The present writer was a mathematics teacher in this school during the whole time of this investigation and was in the position of organizing the mathematics syllabus to suit the investigation. Grades 5 and 6 had the ordinary teaching of Arithmetic. It was similar in all classes with the exception of a two months' period in Grade 6, when an experiment was carried out. This was included in the pilot study arranged for this study. The teaching in Grade 6 and the results of this pilot study have been reported previously (Malinen, 1961).

After Grade 6 the pupils had completed their course in Arithmetic, and they began their course in Algebra at the beginning of Grade 7. The teaching of Geometry began three months later and after that pupils had on the average two lessons per week of Algebra and two lessons per week of Geometry. I have concentrated on studying the teaching of Algebra. The teaching of Geometry was almost identical in all classes and there were few direct connections between Geometry and Algebra because of the experimental situations. There was the same teacher in Algebra and Geometry. We may suppose that the teaching of Geometry has not disturbed the performances in Algebra by causing more differences between pupils and between classes. There were no essential differences between these classes in other subjects either and we assume that other teachers did not produce uneven effects on the learning results in Algebra on the whole.

The experimental period in Algebra began at the beginning of Grade 7 (September, 1961) and lasted three school years, ending in May, 1964. During the whole of this time the pupils had the same mathematics teacher, Mrs. Riitta Timonen in classes B and D, and the present writer in classes A and C. There were two courses in Algebra for these pupils and the subjects were divided into two groups:

Group 1: classes A and B
Group 2: classes C and D.

Hence, both teachers were teaching in both groups and the individual differences between

the teachers would affect both groups in the same way. It was planned that both teachers should use the same teaching methods in both groups so that the teaching progressed mainly according to the typical class teaching methods. However, we might think that the teachers favoured one of their classes or suited themselves to the teaching in one class better than in another. This causes a systematic bias in results. We cannot estimate this effect (Hawthorne effect) objectively, but it should be small according to the comparison of the results in different classes.

Group 1 studied the experimental course, which was planned by the present writer. The text material was given in Grades 7 and 8 to the pupils in the form of mimeographed sheets. These are on file in the Pedagogical Library of Alppilan yhteislyseo and in the Institute of Education, University of Helsinki. Later on, this material was printed in a more developed form as an exercise book (Malinen, 1964). At the same time Group 2 studied the traditional course of Algebra, using the text book of Väisälä (1960). In Grade 9 Group 1 also used this text book, but not in the same way as Group 2.

A more detailed analysis of the courses is in Appendix A. We have not recorded all the information the pupils received in Algebra during these three years, and thus we cannot know the real differences between these two courses. However, we have gathered all the written exercises from some pupils in every class during this time, and in this way we can evaluate the content of these courses. It is the writer's opinion that this is a more precise method than reporting the minutes used for each topic, because the time was not the same for all pupils, especially in home exercises. There are only slight differences in the number of written exercises between the pupils in the same experimental group. We make the assumption that we can describe the given information, especially the differences between the two courses, by means of the number of written exercises.

Tests and Test Administration

We have measured the learning of Algebra by using 20 tests of separate topics of the course. The correspondence of topics and tests can be seen in Appendix A, and some examples of test items are presented in Chapter 3. Most of these tests are ordinary achievement tests, where it is required to solve equations or simplify algebraic expressions. Some tests should measure the understanding of algebraic principles and the ability to apply algebraic knowledge to new situations. Though we have tried to measure different levels of aims, the higher levels of achievement, in particular analysis and evaluation, remained outside the scope of the testing.

Some of the achievement tests were presented to the pupils for the first time in Grade 6 during the pilot study. These tests were connected with the arithmetic course, but they are of interest also for the teaching of Algebra. All algebra tests were presented after the pupils had learned the corresponding topics. For many tests there was further retesting after the pupils had learned more algebra. All tests were presented again in Grade 9 during the final phase of the experiment. These tests did not affect the school marks apart from tests 'Algebra XIV' and 'Problems III' in Grade 9. The pupils did not know the results of the other tests. The pupils were advised to take the tests as exercises which they must perform with care. Besides these tests, the pupils had almost every month an ordinary examination, which included some items of a similar nature to the test items.

In the pilot study some important variables of mathematical ability were formed. When planning these the monographs of Werdelin seemed to give the best basis (Werdelin, 1958; 1961). Therefore I have taken some of his ability tests, which were highly loaded in the numerical, reasoning and deductive factors. These tests were presented at the beginning and at the end of the experimental period in Grade 6. In Grade 7 three tests of visual ability were also included in the test battery. All these were retested in Grade 9. Then the test battery was again widened with three other tests, which were not studied earlier in corresponding situations.

The domain of abilities, especially of mathematical abilities is nowadays well investigated by means of factor analysis. We know the factorial structure of many tests, but we cannot be sure that they function in the same way now, because our pupils were so young, especially in Grades 6 and 7. We cannot be sure that these

tests measure some crystallized abilities, as in a sample of adults.

In the affective domain we are primarily interested in attitudes. In this field there were few studies when this investigation was started. Werdelin's extensive study (1960) was of great help. He carried it out at the University of Illinois, USA, and it was connected with studies of modern mathematics teaching. We have used some of his factor tests, but we cannot be sure that these tests have the same structure in a different school environment. Thus, we must re-analyse them, and to help us in the analysis of the structure of this domain we have included in the test battery other attitude tests as well. Most of them are connected directly with the learning of algebra and we have not tried to study the whole area of the pupils' attitudes. Then, these measurements are not connected firmly with the total systems of attitudes. Neither are they connected with the testing of emotional discrepancies (Magne, 1967).

All tests and their factorial structure will be presented in Chapters 3 and 4. The present writer gave the achievement and ability tests to the pupils, and two students of psychology gave the attitude tests. In a general instruction to the ability and attitude tests the pupils were told that the data were for scientific purposes only and would not be handed over to their teachers. All pupils seemed to be active in the test situations, though they were left uninformed of the results of most tests. The pupils were positively motivated towards the testing, because the tests were presented during the mathematics lessons and this kind of work was pleasant as a change.

In Grade 6 the pupils were tested for about 6 hours, in Grade 7 for about the same amount, in Grade 8 for only 1–2 hours, but in Grade 9 for more than 10 hours. However, this was spread over a long period and did not cause any disturbance in the ordinary school work.

Often some pupils were absent from school during the testing. As a rule, they were tested when they came back to school. In some cases the results have been deduced from the other results. Differences in the time of testing did not cause difficulties, because the other pupils did not get the results of the tests and they had no notes about the items. In this way it was possible to get scores for all pupils in all tests.

The statistical treatment of the test material has been made in several phases and at several centres: Factor analyses with Varimax rotation at the Computer Centre, University of Helsinki (J. Torppa), factor analyses with cosine rotation at the Finnish Cable Company (Anna-Riitta Niskanen), transformation analyses at the Institute of Nuclear Physics, University of Helsinki (T. Hirvonen), and regression analyses at the Computer Centre, University of Jyväskylä (O. Ylinentalo).

Chapter 3. THE FIELD OF ABILITY AND ACHIEVEMENT AND ITS STRUCTURE

It is difficult to separate precisely ability tests and achievement tests before the experimental analysis. All these tests have the same technical form and they also have many similarities as to their contents. So, it seems natural to analyse their structure together. For the preliminary description, we will take as achievement tests those tests which measure achievement in Algebra. All the others are ability tests.

We give here a short description of our tests. The text in the examples presented here is translated from the Finnish original. All these tests, their instructions and the distributions of the scores are on file in the Pedagogical Library of Alppilan yhteislyseo and in the Institute of Education, University of Helsinki.

Ability Tests

We used already in the pilot study some of Werdelin's ability tests. In the report of this study (Malinen, 1961) they have been grouped as reasoning tests and numerical tests. The reasoning tests which have been used in the present study have previously been loaded in the *Reasoning Factor* (Werdelin, 1961, p. /1) or in the *Deductive Factor* (Werdelin, 1958, p. 124) in the extensive analyses of mathematical abilities. The factors in the former analysis have been checked by transformation analysis (Werdelin, 1966 b). These reasoning tests are:

Test No. 1. Number Series. Published by Werdelin (1958, p. 289) under the name Number Series I. 16 items. Examples:

2) 1 2 4 8 16 — —
10) 1 1 2 6 24 120 — —

Test No. 2. Syllogisms. Published by Werdelin (1958, p. 305) under the name Syllogisms I. 21 items. Example:

12) Petri is taller than Lauri and Lauri is smaller than Esko. Is Petri smaller than Esko?

Test No. 3. Arithmetic. Published by Werdelin (1958, p. 303) under the name Arithmetic II. 11 items. Example:

4) Two numbers have a relation 4 : 7 and their sum is 44. What are the numbers?

In the final battery in Grade 9 we have used a test constructed by the present writer. This is a nonverbal form of a typical reasoning test:

Test No. 4. Comparisons. 12 items. A short cut instruction: Write $>$, $=$, $<$ or ? between the letters in the answer. Examples:

2)	$B < C$	Answer:		
	$A > C$		A	B
10)	$B = C$	Answer:		
	$A > D$		A	C
	$B = D$			

We have used three of Werdelin's numerical tests. These are:

Test No. 5. Addition I. Published by Werdelin (1958, p. 313) under the name Addition I. 108 items. Examples:

3	7
1	4
+4	+8

Test No. 6. Addition II. Published by Werdelin (1958, p. 325) under the name Addition III. 87 items. A short cut instruction: Indicate a correct answer by ./. and a wrong one by V. Examples:

5	8
6	9
8	6
8	4
7	5
24	32

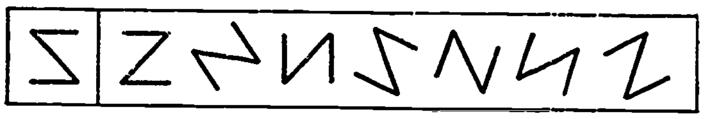
Test No. 7. Multiplication. Published by Werdelin (1958, p. 315) under the name Multiplication. 66 items. Examples:

$$\begin{array}{r} 641 \\ \cdot\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 699 \\ \cdot\ 4 \\ \hline \end{array}$$

In Grade 7 we have augmented our battery of tests with three visual tests constructed by Werdelin:

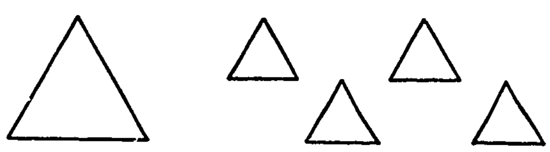
Test No. 8. Figures. Published by Werdelin (1958, p. 326) under the name Figures. 21 items. A short cut instruction: Write "V" under the turned-around figures. Example:

6)



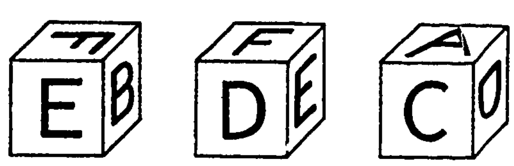
Test No. 9. Form Boards. Published by Werdelin (1958, p. 331) under the name Form Boards II. 24 items. A short cut instruction: Divide the figure to obtain the smaller figures. Example:

5)



Test No. 10. Cubes. Published by Werdelin (1958, p. 320) under the name Cubes I. 12 items. A short cut instruction: Here are three pictures of the same cube. There are the letters A, B, C, D, E, and F written on this cube. Write the opposite letters. Example:

4)



Answer:

___ and ___ ___ and ___ ___ and ___

In the final battery of tests we have used two further tests constructed by the present writer. These should measure the learning of new mathematical concepts. Because the pupils did not understand the content of these tasks at all, a detailed instruction was needed which took almost as long as the test itself. These tests are:

Test No. 11. Operations. 12 items. The instruction took about 4 minutes. First the meaning of the operation in general was presented. Then an operation $a \circ b = 2a + b$ was introduced by means of three examples. The test included exercises where this operation was used. Examples:

2) $1 \circ 3 =$ 8) $(4 \circ 0) \circ (2 \circ 1) =$

10) Solve the equation $3 \circ x = 11$.

Test No. 12. Finite Arithmetic. 12 items. The instruction took about 4 minutes. The test leader first drew a dial and said that $11 + 3 \equiv 2$ (mod 12). Four other examples were presented in mod 12. The test included exercises in mod 12 and mod 5. Examples:

2) $17 \equiv$ ___ (mod 12)

8) Solve the equation: $x + 5 \equiv 8$ (mod 12

$x \equiv$ ___ (mod 12)

11) $13 + 9 \equiv$ ____ (mod 5)

There was one more test which we have included in this group. This has been taken from Heikkinen's work curve investigations (Heikkinen, 1952):

Test No. 13. Work Test. This test includes many columns of simple additions and the sum of two consecutive numbers in a row is to be written between these numbers (tens omitted), as in the following example:

$$\begin{array}{cc} 1 & \\ & 5 \\ 4 & \\ & 7 \\ 3 & \\ & 2 \\ 9 & \end{array}$$

After three minutes the pupils draw a line under their last addition. We took 14 periods of 3 minutes each, i.e. a total testing time of 42 minutes. This test has been used for many purposes. We have here used only the number of additions per minute and call this *Variable No. 13. Work Test Addition.*

The testing times and the scoring formulas are found in Table 3.1. For the tests Nos. 1–12 the test variable has been formed by adding the scores. The means and standard deviations (S.D.) for these variables are also presented in Table 3.1.

Table 3.1. Data on the Ability Tests (N = 119)

No.	Test	Testing time (min) Grade 6 Grade 7	Testing time (min) Grade 9	Grade 6 Grade 7 Mean	Grade 6 Grade 7 S.D.	Grade 9 Mean	Grade 9 S.D.	Scoring formula
1	Number Series	12	8	8.1	2.8	9.9	2.1	R
2	Syllogisms	10	6	11.2	4.6	12.6	5.4	$R-\frac{F}{2}$
3	Arithmetic	25	15	5.4	1.8	6.4	2.3	R
4	Comparisons	–	5	–	–	6.1	2.7	$R-\frac{F}{3}$
5	Addition I	5	3	92.6	13.8	77.4	15.0	R
6	Addition II	7	4	40.3	11.8	30.0	9.3	$R-F$
7	Multiplication	6	4	30.6	7.5	25.4	6.7	R
8	Figures	6*	6	9.6	4.2	13.4	4.2	$\frac{R-F}{7}$
9	Form Boards	6*	6	11.3	3.0	13.0	3.5	R
10	Cubes	10*	10	2.9	3.2	5.3	4.1	R
11	Operations	–	5	–	–	7.2	3.3	R
12	Finite Arithmetic	–	5	–	–	8.1	2.5	R
13	Work Test Addition	–	42	–	–	33.1	7.3	–

* The test was presented in Grade 7.

The testing times have been decreased in the study in Grade 9 for many tests and then they were the same as in Werdelin's study. It was necessary to take longer times at first, because the pupils were very young in Grade 6. It was difficult to guess suitable testing times, because the training effect was not easy to foresee. The distributions of the scores were, however, almost normal for most tests. The greatest deviations were in the test 'Addition I' in Grade 6 during the second testing, when many pupils reached the end of the test (ceiling effect). The opposite situation occurred in the test 'Cubes' in Grade 7, when a lot of pupils did not get any items solved (floor effect). Nevertheless, these variables have the same properties as the others as to reliabilities and communalities. We suppose that all these variables have interval scales.

Table 3.2. Coefficients of Reliability and Constancy of the Ability Tests

No.	Test	Coefficients of reliability: Split-half method	Coefficients of reliability: Retest or parallel test method	Coefficients of constancy (N = 119): Between two testings in Grade 6, interval 2 months	Coefficients of constancy (N = 119): Between testing in Grade 6 (7) and Grade 9, interval 3 (2) years
1	Number Series	.67–.75	–	.66	.50
2	Syllogisms	.90–.95	–	.61	.72
3	Arithmetic	.45–.70	–	.60	.60
4	Comparisons	.68–.75	–	–	–
5	Addition I	–	.78	.78	.75
6	Addition II	–	.75	.75	.69
7	Multiplication	–	.81	.81	.64
8	Figures	–	.71	–	.71
9	Form Boards	.79–.91	–	–	.68
10	Cubes	.93–.96	–	–	.74
11	Operations	.94–.98	–	–	–
12	Finite Arithmetic	.66–.72	–	–	–
13	Work Test Addition	–	.69	–	–

We have estimated the reliabilities for most tests by using the split-half method. The coefficients (Pearson's r-coefficients) have been calculated for each class separately. The highest and lowest values found are in Table 3.2. When calculating the constancy coefficients in the last column we have used the later of the two testings in Grade 6 of the reasoning and numerical tests.

For our purpose it would be useful to combine test variables where it seems suitable. The greatest parallel test coefficient, .75, is between the tests 'Addition I' and 'Addition II' in Grade 9. Furthermore, they have the same task (additions). We have combined them by dividing the raw scores by the standard deviation of each test and then adding the scores. In this way we have obtained the variable 'Addition I, II'.

According to the writer's subjective analysis, the tests 'Finite Arithmetic' and 'Operations' should measure the ability to learn new topics in mathematics. The pupils had not performed any items of these tests before the instruction. The performance after the instruction is a result of the pupils' ability to understand and apply the information rapidly. We cannot be sure if these tests measure a crystallized ability (c.f. page 13) and this cannot be verified, as these tests cannot be presented twice. The correlation coefficient between these tests is only .29 (N = 119). The reliabilities of these tests in Table 3.2. were much higher. This gives rise to doubt whether we have measured a common ability to learn new mathematical things rapidly. Thus, we cannot combine these test variables. For further investigation we have taken the test 'Operations', because it has a greater reliability.

Achievement Tests

The testing of achievements began already in Grade 6 during the pilot study. Then, the tests 'Equations', 'Problems I' and 'Problems II' were constructed by Mr. K. Virtanen and the present writer. During the experimental period in algebra, 17 other achievement tests were constructed by the present writer. In most cases the instruction for these tests was easy: "Solve the equations" or "Simplify the expressions".

Test No. 14. Equations. Simple equations, mostly cases of the model $ax + b = c$, where a, b, and c are rational numbers. This is almost analogous with the test 'Equations I' by Werdelin (1958, p. 316). Two parallel tests of 24 items each given in Grades 7 and 9. Examples:

10) $2{,}3 \cdot x - 3 = 1{,}6 \qquad x =$ _____

15) $\dfrac{x}{11} - 28 = 3 \qquad x =$ _____

Test No. 15. Algebra I. Simple expressions with the addition or subtraction of positive and negative numbers. 20 items. Two parallel tests. Examples:

6) $(+3) - (-3\tfrac{1}{2}) =$ 18) $13 - (6 - 7) =$

Test No. 16. Algebra II. Simple expressions with multiplications or divisions of positive and negative numbers. 20 items. Two parallel tests. Examples:

7) $-11 \cdot (-1) =$ 14) $-\dfrac{-50}{5} =$

Test No. 17. Algebra III. Complex expressions combined from previous operations. 18 items. Examples:

7) $\dfrac{-25 + 15}{-6\frac{1}{2} \cdot 4} =$

14) $3\tfrac{2}{3} + (-3)(+4\tfrac{1}{3}) + 7 =$

Test No. 18. Algebra IV. Calculating the numerical values of algebraic expressions. 12 items. Example:

7) What is $\dfrac{1-a}{1-b}$, where $a = 6$ and $b = -5$?

Test No. 19. Algebra V. Identification of six mathematical principles in situations where some expressions are simplified. The principles are: commutative and associative laws for addition and multiplication, distributive laws for

multiplication and division. 10 items. A short cut instruction: Mark the number of the principle. Example:

6) $8x \cdot (5y - 2) =$

$= 8x \cdot 5y - 8x \cdot 2 =$ ____

$= 8 \cdot 5 \cdot xy - 8 \cdot 2 \cdot x =$ ____

found by calculation

$= 40xy - 16x.$

Test No. 20. Algebra VI. Algebraic expressions including the addition of terms with the same variable. 20 items. Examples:

8) $-b - 3 + 4b =$

16) $9y + (5x - 2y) - 3y =$

Test No. 21. Algebra VII. Algebraic expressions including additions and multiplications. 12 items. Examples:

5) $(3a + b)(3a - b) =$

10) $-3a(2a - 3) + 2a(3a + 5) =$

Test No. 22. Algebra VIII. Simple rational algebraic expressions. 18 items. Examples:

5) $\frac{2x}{3} \cdot \frac{7}{3} =$ 14) $\frac{a+b}{ab} \cdot 2 \cdot (-a) =$

Test No. 23. Algebra IX. Exponential expressions. 20 items. Examples:

3) $(a^3bc^4)^2 =$ 14) $\left[(a^m)^n\right]^p =$

Test No. 24. Algebra X. Rational algebraic expressions (of medium difficulty). 18 items. Examples:

5) $\frac{1}{3a} - \frac{1}{2a} =$ 12) $(3a - 6) : \frac{a}{2} =$

Test No. 25. Algebra XI. Equations (of medium difficulty). 14 items. Two parallel tests. Examples:

5) $4x - x - 5 = 8$ 11) $2{,}5 + \frac{2x}{3} = -4{,}5$

$x =$ $x =$

Test No. 26. Algebra XII. Rational algebraic expressions (of high difficulty). 7 items. Example:

5) $1 - \frac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}} =$

Test No. 27. Algebra XIII. Difficult equations. 7 items. Example:

5) $\frac{2}{x-1} - \frac{1}{x-2} = \frac{1}{x-3}$ $x =$

Test No. 28. Algebra XIV. Use of coordinate system and functions. 6 items. Example:

3) Investigate by means of the graphic method whether the point $(-1, -3)$ lies on the line $y = 2x + 1$.

Test No. 29. Problems I. Solution of two types of problems: "What proportion of a is b?" and "What is $\frac{m}{n}$ of a?" 12 items. Examples:

2) During the journey 40 kgs. of the total 440 kgs. of goods disappeared. What proportion remained?

9) If Jussi had been 3 centimetres taller, his height would be $\frac{5}{4}$ of Heikki's height, which is 1 metre 24 centimetres. How tall is Jussi?

Test No. 30. Problems II. Applications of the linear relationship between two variables. 10 items. Example:

2) Riding his motorcycle 50 kilometres has taken Matti 2 hours. How long did the rest of the journey, which is 20 kilometres, take if the speed was as before?

Test No. 31. Problems III. Different kinds of difficult problems where the solution calls for the use of equations. 6 items. Example:

4) In a money-box there were only 20-penni and 50-penni coins. There were 17 coins altogether and their total value was 6.70 marks. How many coins were there of each kind?

Test No. 32. Understanding. This test was constructed on the basis of Saad's monograph (1960). There are 11 items measuring the understanding of concepts and principles, but item No. 8 is excluded because of its slight internal consistency. Furthermore, there are 15 items measuring the knowledge of concepts, but we have not used them. The former 10 items form

the test 'Understanding of Algebra' or simply 'Understanding'. Example:

7) We get a^2 from the division $\frac{a^5}{a^3}$. Why?

___ 1. Because $5a - 3a = 2a$.

___ 2. Because $\frac{\cancel{a}+\cancel{a}+\cancel{a}+a+a}{\cancel{a}+\cancel{a}+\cancel{a}} = a^2$

___ 3. Because it is possible to subtract indices in those cases.

___ 4. Because $\frac{\cancel{a}\cdot\cancel{a}\cdot\cancel{a}\cdot a\cdot a}{\cancel{a}\cdot\cancel{a}\cdot\cancel{a}} = a^2$.

___ 5. For none of these reasons.

Test No. 33. Algebraic Sentences. Conclusions of the form "If . . ., then . . .". 10 items. Example:

4) If $b > 0$, then $b - 2 > 0$.

always true	sometimes true	always false

In addition, there are 5 items which involve proofs of algebraic sentences, but we have not used these items because there were mostly zero scores.

The scoring of these tests was obtained generally by using the scale of 0 and 1 points. In tests Nos. 26–30 the score $\frac{1}{2}$ was also used, when the idea was correct but there was a computational error. In the total score a half is raised to the nearest higher integer. Tests Nos. 28 and 31 are scored with the points 0, 1, and 2. There was a correction for guessing in test No. 33, but not in the other tests. In this way we have obtained a variable for each test, which we call by the same name.

Tests Nos. 14, 15, 16, and 25 had two parallel forms and we have used the means of the scores on these parallel tests.

Testing times, measures of reliability and constancy coefficients are presented in Table 3.3. The time-table for the testing is in Table 6.7. The reliability coefficients have mostly been computed for each class separately. The constancy coefficients are for the whole group and between two consecutive presentations. The time intervals can be found in Table 6.7. The testing times were shortened in Grade 9, since the pupils already had a good routine in handling simple expressions and equations. In spite of that, they still had the nature of power tests, except for tests Nos. 14, 15, 16, and 22,

Table 3.3 Data on the Achievement Tests

		Testing time (min)		Coefficients of reliability		Coefficients of constancy (N = 119)
No.	Test	Grade 6 Grade 7	Grade 9	Split-half method	Parallel test method	
14	Equations	10	8	–	.66–.85	.66 .44
15	Algebra I	7	5	–	.79–.86	.62 .65
16	Algebra II	7	5	–	.74–.86	–
17	Algebra III	15	12	.68–.82	–	.70 .62
18	Algebra IV	15	12	.67–.76	–	.09
19	Algebra V	15	10	.72–.92	–	.57
20	Algebra VI	8	6	.86–.95	–	.13
21	Algebra VII	15	10	.48–.54	–	.49
22	Algebra VIII	7	5	.71–.73	–	.25
23	Algebra IX	15	12	.76–.91	–	.61
24	Algebra X	20	15	.66–.82	–	.53
25	Algebra XI	15	10	.72–.91	–	.48
26	Algebra XII	–	25	.59–.95	–	–
27	Algebra XIII	–	35	.59–.69	–	–
28	Algebra XIV	–	40	.72–.73	–	–
29	Problems I	30	25	.82	.43–.48	.67 .60
30	Problems II	30	25	.28–.53	.43–.48	–
31	Problems III	–	40	.45–.62	–	–
32	Understanding	–	20	.37	–	–
33	Algebraic Sentences	–	18	.54	–	–

where in many cases the time ended before the skill.

The distributions of scores have in this way remained suitable, even if the ceiling effect was apparent, especially in tests Nos. 15, 16, and 20. There was, however, no essential difference in reliability coefficients between Grades 7 and 9. It is still possible to assume that all these test variables have interval scales.

There are some low reliability coefficients, especially for tests Nos. 29–33. It may be that the achievement in such tests is not very coherent and the tests have low internal consistency. The number of items in these tests is also rather small. There may be changes in the structure of performances in those tests, which have low or medium constancy coefficients.

Because many of these tests do not differ much from each other as to their content, we will investigate possibilities of combining them for structure analysis. Some most important correlation coefficients between tests can be seen in Table 3.4. The coefficients are mostly separate for each class in Grade 9.

Table 3.4. Correlation Coefficients between the Achievement Tests

Algebra I–II	.79
Algebra III–IV	.75
Algebra VI–VII	.52–.67
Algebra VII–IX	.40–.44
Algebra VIII–X	.53–.61
Algebra IX–X	.35–.49
Algebra X–XII	.66–.69
Algebra XII–XIII	.59–.63
Problems I–II	.59–.70

There are some coefficients in Table 3.4, which are as high as the reliability coefficients of the tests. We will combine the test variables 'Algebra I' and 'Algebra II' into a variable 'Algebra I, II' by adding the raw scores. In the same way we will combine the variables 'Problems I' and 'Problems II' into a new variable 'Problems I, II'. The combination of variables 'Algebra III' and 'Algebra IV', like the combination of variables 'Algebra XII' and 'Algebra XIII', has been made by weighting the scores by the inverses of the standard deviations.

Most algebra tests measure comprehension and application without any precise differentiation between these. Problem tests are perhaps pure application tests. The test 'Algebra V' measures comprehension purely. The test 'Understanding' includes much analysis and the test 'Operations' both analysis and synthesis. The test 'Comparisons' can also be evaluated as the analysis of relations. So, these three tests are in Bloom's taxonomy at the highest level when compared to the others. However, these tests include the understanding of simple principles, which are essential for elementary algebra. We have not formed variables on the lowest levels in Bloom's taxonomy, because it is difficult to find essential differences there. Our measurements include the typical cognitive outcomes concerning these learning processes. Though we have not measured all topics of elementary algebra these variables have all together a good content validity concerning the curriculum.

Structure in Grade 7

We will investigate the structure of the tests which were presented to all pupils in Grade 6 or in Grade 7. We have included in the test battery a variable 'Mark in Algebra'. This mark in Algebra was given to all pupils at the end of the academic years, using the scale from 4 to 10. It is an estimate of the school success in algebra during this academic year. This variable was almost normally distributed. Thus, we have obtained the following variables:

No.	Variable	Time of presentation
1	Number Series	February 1961
2	Syllogisms	–„–
3	Arithmetic	–„–
4	Addition I, II	–„–
5	Multiplication	–„–
6	Figures	April 1962
7	Form Boards	–„–
8	Cubes	–„–
9	Equations	September 1961

10	Algebra I, II	October 1961
11	Algebra III	—„—
12	Algebra V	November 1961
13	Algebra VIII	April 1962 (Group 1) November 1962 (Group 2)
14	Problems I, II	September 1961
15	Mark in Algebra	May 1962

This battery includes both ability and achievement test variables and its structure will be described most naturally by factors, which are not pure ability factors. We have not concentrated here on using only ability factors (like R and N factors) which can be determined beforehand by the known factor tests, because these do not perhaps describe well the differences in this situation. There is more than one year between the first and the last tests that were given. It may be that some factors are already connected with the fact that the tests were presented at different times. Nevertheless, we take this as a cross-section of abilities and achievements.

We have analysed this battery of variables by means of factor analysis. The correlation matrix and the centroid factor matrix are in Appendix B (Tables 3.5 and 3.6). The eigenvalues in the factor tables have been computed according to Harman (1960). We have omitted the zero integers and the decimal points in all correlation and factor matrix tables. For the rotation of the centroid factor matrix we have taken only four factors, because the eigenvalues of factors were small after that. We have used here the Varimax rotation method, and the rotated factor matrix is shown in Table 3.7. We have made an attempt to rotate also with five factors, but the last factor turned out to be a factor specific to a single test.

Table 3.7. Varimax-rotated Factor Matrix, Ability and Achievement Tests, Grade 7

No.	Variable	Factors I	II	III	IV	h^2
1	Number Series	*59*	02	31	−15	47
2	Syllogisms	*40*	−09	38	23	37
3	Arithmetic	27	06	*60*	22	49
4	Addition I, II	18	*80*	10	07	68
5	Multiplication	11	*83*	04	06	70
6	Figures	24	10	*56*	−08	38
7	Form Boards	06	03	*54*	11	31
8	Cubes	35	15	*38*	13	31
9	Equations	41	18	*53*	33	59
10	Algebra I, II	*74*	24	15	29	72
11	Algebra III	*71*	22	19	39	74
12	Algebra V	*51*	16	25	11	36
13	Algebra VIII	14	09	09	*57*	36
14	Problems I, II	26	−01	*48*	46	51
15	Mark in Algebra	*66*	15	27	42	70

Interpretation of factors (The largest loading of each variable has been printed in the tables in italics.):

Factor I. *Algebra and induction factor.* The highest loadings are in tests where there are simple expressions. These are still important also for the mark in Algebra.

Factor II. *Numerical factor.* It is important to point out that the solving of simple equations and the simplifying of simple expressions is not closely connected with this factor in Grade 7.

Factor III. *Visual-reasoning factor.* In this situation no separate factors for reasoning and visual abilities have been obtained. The performance of visual tests, especially of the test 'Cubes', still calls for much reasoning ability, as does the test 'Equations', also.

Factor IV. *Factor of complex Algebra.* The highest loading is in variable 'Algebra VIII', which includes newly learned material.

We will discuss more about the content of these factors after the comparisons of factor structures. A detailed analysis of the most important factors will be made in Chapter 5.

Structure in Grade 9

The same tests as presented before were also in the final testing in Grade 9, which took place mainly during December 1963 and January 1964. The 'Mark in Algebra' was now given in May 1964. We have first analysed these common variables in the new testing situation.

The correlation matrix and the centroid factor matrix are presented in Appendix B (Tables

3.8 and 3.9). Also here four factors were taken into the rotation, which was made as in Grade 7. Its results are in Table 3.10.

Table 3.10. Varimax-rotated Factor Matrix, Common Ability and Achievement Tests, Grade 9

No.	Variable	Factors I	II	III	IV	h^2
1	Number Series	40	18	16	*55*	52
2	Syllogisms	*52*	19	17	42	51
3	Arithmetic	30	03	27	*55*	47
4	Addition I, II	11	*74*	15	10	59
5	Multiplication	12	*72*	05	04	54
6	Figures	*57*	14	02	25	41
7	Form Boards	20	04	15	*57*	39
8	Cubes	*64*	04	22	13	47
9	Equations	05	*48*	33	25	41
10	Algebra I, II	22	13	*63*	35	59
11	Algebra III	14	20	*60*	43	59
12	Algebra V	06	18	35	*54*	45
13	Algebra VIII	15	15	*77*	08	64
14	Problems I, II	34	15	30	*57*	56
15	Mark in Algebra	11	12	*71*	38	68

Interpretation of these factors:

Factor I. *Visual-reasoning factor.* This factor has fewer high loadings than the visual-reasoning factor in Grade 7. Visualisation was important also in 'Syllogisms', because many pupils had drawn graphs of the situations.

Factor II. *Numerical factor.* The loading of the test 'Equations' is now much larger than in Grade 7, but the loadings of the algebra tests have not increased.

Factor III. *Factor of school success in Algebra.* The highest loading is now in the test 'Algebra VIII'. This test is now loaded in the same factor as the tests 'Algebra I, II' and 'Algebra III', all being closely connected with the 'Mark in Algebra'. Instead of this, the test 'Algebra V' is no longer highly loaded in the same factor as the other algebra tests.

Factor IV. *School reasoning factor.* This is not completely identical to the reasoning factor in Grade 7.

We will now investigate the structure of ability and achievement tests on a wider basis. In Grade 9 there were more tests than before and we have submitted most of them to a new analysis. We have omitted some algebra tests, because there are several tests which resemble each other as to their structure. Thus, this battery of test variables can be used to investigate the structure of the field of all ability and achievement tests. The variables of this analysis are shown in Table 3.13. The correlation matrix and the centroid factor matrix are in Appendix B (Tables 3.11 and 3.12). We have used five factors in the rotation which was made by means of the analytic cosine rotation method presented by Ahmavaara and Markkanen (Vahervuo & Ahmavaara, 1958; Markkanen, 1964). The best two solutions according to the criterium presented in the rotation programme were taken and the better of these is published in Table 3.13.

Table 3.13. Cosine-rotated Factor Matrix, All Ability and Achievement Tests, Grade 9

No.	Variable	Factors I	II	III	IV	V
1	Number Series	17	*64*	17	−27	36
2	Syllogisms	28	*62*	22	−51	*62*
3	Arithmetic	13	37	17	−22	*64*
4	Addition I, II	*86*	00	−00	00	00
5	Work Test Addition	*80*	−05	−10	04	26
6	Multiplication	*66*	−01	−16	36	−26
7	Comparisons	05	42	15	−19	*64*
8	Figures	10	*54*	46	−57	33
9	Form Boards	00	*67*	00	−00	00
10	Cubes	−01	*47*	42	−33	31
11	Operations	17	07	21	−10	*59*
12	Equations	41	09	−20	*60*	−19
13	Algebra I, II	01	−01	*50*	42	11
14	Algebra III	07	10	23	30	*45*
15	Algebra V	21	18	03	25	*26*
16	Algebra VI	00	00	*69*	00	00
17	Algebra IX	00	00	00	*69*	00
18	Algebra XI	10	−10	20	*75*	−13
19	Algebra XII, XIII	−03	−30	23	*85*	06
20	Problems I, II	12	*61*	−02	07	22
21	Problems III	−18	−05	21	*68*	−03
22	Understanding	−00	00	−00	00	*63*
23	Mark in Algebra	−03	−07	28	*70*	10

The analytic cosine rotation is an oblique rotation. The variables Nos. 19, 21, and 23 have been passive in the rotation and they cannot be base vectors. We have overlooked them here, because they are later used as criterion variables. We have made the rotation also with the Varimax method (unpublished) and it gives almost identical factors. The factors in this oblique rotation were, however, easier to interpret.

Interpretation of factors:

Factor I. *Numerical factor.* It is interesting to note that the variable 'Work Test Addition' has a high loading in this factor. This test took 42 minutes and the test 'Addition I' only 3 minutes. The testing time is of no importance to the factor structure. Both tests have been solved using the same automatised rule and it gives them the same factor structure. This is in line with Werdelin's theory concerning the numerical factor (Werdelin, 1958, pp. 166–193). The differences between pupils are caused by fluency in working but not by tenacity in working. The test 'Equations' also has a considerable loading in this factor.

Factor II. *Visual-reasoning factor.* There are high loadings in the same variables as in factor IV in the reduced factor analysis for Grade 9 (Table 3.10), excluding the algebra tests. This is now a "pure" ability factor without loadings in school variables.

Factor III. *Algebra and visual factor.* This is between factors II and IV without any special characteristics.

Factor IV. *Factor of school success in Algebra.* This is almost like factor III in the reduced analysis (Table 3.10), but there is now greater weight in the difficult tests.

Factor V. *Factor of understanding mathematical principles.* The tests 'Comparisons' and 'Operations' were unfamiliar to the pupils and we can suppose that this includes also the ability to learn new things in mathematics.

Table 3.14. Intercorrelations of Factors in Table 3.13

	I	II	III	IV
II	16			
III	39	35		
IV	23	64	40	
V	−01	25	01	61

There is a high correlation between factors II and IV, as also between factors IV and V in Table 3.14. Thus, the reasoning factor and the understanding factor are connected with the factor of school success in Algebra more than the achievement factors on the whole. It is important to state that our understanding factor and reasoning factor have only a slight correlation (.25). Reasoning is connected more with trained operations as those in 'Problems I, II'. Reasoning is a crystallized ability factor, understanding is a situational factor, which is less important when the learning process is complete.

It was our problem to investigate whether we can measure different levels of understanding. According to this result, it is possible to form a factor which gives a partial solution to this problem. Pupils who are high in the factor of understanding can understand and use new principles in mathematics, but pupils who are low in this factor, not so well. However, we cannot be sure that we have separated different levels of understanding in this way so that these levels could correspond with the levels in Gagné's presentation (1965).

Comparison between the Factor Structures

We have used the same tests in the factor analyses in Grade 7 and in Grade 9. We will now compare these structures by rotating one structure into another, which can be done in more than one way.

There are the same subjects in both analyses and this gives us the possibility of proceeding in the manner that Werdelin (1962, pp. 196–204) has presented in his article concerning the synthesis of factor analyses. If we accept that we can take into the same combined battery tests which have been presented during a long interval of time, we can use the development analysis presented by Heinonen (1967). Then, no comparison between the factor structures of Grade 7 and Grade 9 is needed. We have used the latter method in the following paragraph.

We can compare the structures of two factor analyses in another way, if these analyses have common tests. In this situation all tests are common when comparing the factors in Tables 3.7 and 3.10. There are, however, differences in test performances between Grades 7 and 9. The test groups in these two situations are not iden-

tical and their difference will be seen in those tests where the teaching or the maturation has had an effect. There are probably many tests where the changes in performances have not been essential. These are the real common tests in this investigation. When transforming one structure into another there will be abnormal transformation in those cases where the test has changed its structure.

If we compare the structures by using the common tests, we do not need the same subjects. In our situation we lose information by using this method, because we do not use the correlations between the variables of different analyses. Instead of that, we make a comparison between the factor structures by transforming the structure in Grade 7 into another. The theoretical background of these comparisons has been discussed by Werdelin (1962, pp. 143–154). We use the symmetric transformation analysis presented by Mustonen (1966).

The transformation matrix L_{12} from one factor matrix A_1 to another A_2 is given by

$$A_1L_{12} = A_2.$$

Abnormal transformation is measured by using the residual matrix

$$E_{12} = A_1L_{12} - A_2$$

and the diagonal of the matrix $E_{12}E'_{12}$, which gives the residuals of the variables. The residuals of the variables are equal when transforming in both directions and $L_{21} = L_{12}^{-1}$. That is why this is called a symmetric transformation analysis.

We have made this transformation analysis between the Varimax-rotated factor matrices which are in Tables 3.10 (matrix A_1) and 3.7 (matrix A_2). The transformation matrix L_{12} is in Table 3.15. Factor II is in both matrices almost identical. Thus, the numerical ability has clearly remained unchanged. Factor III in Grade 7 and factor I in Grade 9 also show a good correspondence and we have interpreted them as visual-reasoning factors. The other factors do not correspond completely with each other. The factor of complex algebra in Grade 7 has changed greatly to the factor of school success in Grade 9. It is obvious that school success in Grade 9 depends on the performance of more complex tasks than in Grade 7.

The residuals of the variables in Table 3.16 are independent of the direction of the transformation. The highest values of the residuals

Table 3.15. Transformation Matrix L_{12}, Common Ability and Achievement Factors

		Factors in Grade 7			
		I	II	III	IV
	I	−0.284	0.023	0.927	0.268
Factors	II	0.129	0.986	0.044	−0.084
in	III	0.437	0.024	−0.114	0.895
Grade 9	IV	0.833	−0.162	0.367	−0.361

Table 3.16. Residuals of the Variables in Transformation

No.	Variable	Residual	No.	Variable	Residual
1	Number Series	0.121	9	Equations	0.264
2	Syllogisms	0.134	10	Algebra I, II	0.123
3	Arithmetic	0.098	11	Algebra III	0.019
4	Addition I, II	0.009	12	Algebra V	0.015
5	Multiplication	0.029	13	Algebra VIII	0.190
6	Figures	0.057	14	Problems I, II	0.028
7	Form Boards	0.230	15	Mark in Algebra	0.076
8	Cubes	0.206			

are in the variables 'Equations', 'Form Boards' and 'Cubes'. The lowest value is in the variable 'Addition I, II'. We have obtained even more information concerning this transformation, e.g. the residual matrix E_{21} and the differences in the length and angle of variables, but we have not needed them here.

The abnormal transformation of the variable 'Equations' can already be foreseen in Tables 3.7 and 3.10. Its loading in the visual-reasoning factor has diminished and in the numerical factor increased during this time. This can be interpreted as automatization of the process and it is caused here by the school work as in the corresponding situation presented by Werdelin (1958, p. 185). The abnormal transformation of the variables 'Form Boards' and 'Cubes' can be caused by many facts. They are loaded in factors which do not correspond completely with each other and the highest loading of the variable 'Form Boards' has shifted to another factor. There may also be changes in the structure of performance and in the technical characteristics of these variables.

According to our analysis there are only two common factors which can be interpreted as ability factors: the numerical factor represented by the tests 'Addition I, II', and 'Multiplication', and the visual-reasoning factor represented by the tests 'Syllogisms', 'Arithmetic',

'Figures' and 'Cubes'. Besides, there is a factor which describes achievement in Algebra. We have called it *Factor of complex Algebra* in Grade 7 and *Factor of school success in Algebra* in Grade 9. The fourth factor in both analyses is connected with reasoning. In Grade 7 it is connected also with many algebra tests (situational factor), but in Grade 9 it is more an ability factor.

The factor structure of a test battery depends on the characteristic of the tests. Here we had few visual tests and they were performed using resoning ability (especially the test 'Cubes'). Thus, in this battery of tests, they did not form a special visual factor. It is also interesting to note that achievements and abilities formed separate factors (except one unstable factor). The factor fission could not be investigated precisely in these circumstances, but we have not observed it here.

Development Analysis

In transformation analysis we have needed the correlation coefficients between the variables in Grade 7 and Grade 9 separately, but not the correlations between the variables in the different grades (e.g. the constancy coefficients), because the transformation analysis can be made by using two different groups of subjects. However, we have the same subjects in different

Table 3.19. Varimax-rotated Factor Matrix with 5 Factors, Development Analysis

No.	Variable	Grade	Factors I	II	III	IV	V	h_5^2	h_5
1	Number Series	6	30	06	04	*57*	−06	42	65
2	—„—	9	32	28	14	*43*	22	43	66
3	Arithmetic	6	19	*49*	10	33	11	40	63
4	—„—	9	26	*62*	−02	15	24	53	73
5	Addition I, II	6	16	06	*82*	11	−07	73	85
6	—„—	9	12	09	*82*	07	09	71	84
7	Figures	7	07	19	02	*68*	27	58	76
8	—„—	9	−01	17	10	*69*	28	59	77
9	Form Boards	7	09	19	05	23	*73*	63	80
10	—„—	9	20	19	−01	13	*76*	68	82
11	Equations	6	29	*40*	38	39	04	53	73
12	—„—	7	35	45	29	*46*	07	62	78
13	—„—	9	45	14	*48*	09	10	46	68
14	Algebra I, II	7	*68*	25	21	28	05	65	80
15	—„—	8	*62*	30	21	27	06	59	77
16	—„—	9	*57*	38	18	12	21	56	75
17	Algebra III	7	*74*	31	23	20	03	74	86
18	—„—	8	*58*	43	14	20	07	58	76
19	—„—	9	*56*	43	20	15	17	58	76
20	Algebra V	7	*49*	02	15	44	02	46	68
21	—„—	9	*50*	16	12	34	22	45	67
22	Algebra VIII	7	28	*38*	07	−03	−11	24	49
23	—„—	9	*73*	06	13	08	07	56	75
24	Problems I, II	6	25	*69*	06	19	26	64	80
25	—„—	7	23	*70*	15	14	33	70	83
26	—„—	9	36	*44*	07	29	39	56	75
27	Mark in Arithmetic	6	51	*56*	10	24	11	65	80
28	Mark in Algebra	7	*80*	33	03	25	09	82	90
29	—„—	8	*78*	36	09	03	22	80	89
30	—„—	9	*76*	32	00	02	25	74	86
	Eigenvalues per variable		.22	.13	.07	.09	.07	.59	

Table 3.20. Varimax-rotated Factor Matrix with 7 Factors, Development Analysis

No.	Variable	Grade	Factors I	II	III	IV	V	VI	VII	h_7^2	h_7
1	Number Series	6	17	08	04	20	01	23	*63*	52	72
2	–„–	9	26	28	15	20	25	02	*53*	55	74
3	Arithmetic	6	17	*50*	09	18	13	24	18	42	65
4	–„–	9	32	*59*	–01	17	20	–10	13	55	74
5	Addition I, II	6	15	06	*83*	06	–08	06	11	74	86
6	–„–	9	13	09	*82*	10	07	07	–03	72	85
7	Figures	7	15	17	04	*76*	17	06	19	71	84
8	–„–	9	05	17	11	*77*	19	15	11	71	84
9	Form Boards	7	08	20	03	20	*74*	14	04	67	82
10	–„–	9	21	19	–00	13	*77*	01	07	69	83
11	Equations	6	22	*42*	35	16	09	40	21	59	77
12	–„–	7	29	*47*	26	23	11	*47*	20	69	83
13	–„–	9	39	13	*48*	–07	14	13	21	48	69
14	Algebra I, II	7	*62*	22	22	07	07	10	44	70	84
15	–„–	8	*61*	26	23	16	05	09	29	61	78
16	–„–	9	*61*	33	20	15	17	03	08	59	77
17	Algebra III	7	*69*	28	24	00	06	19	32	75	87
18	–„–	8	*60*	39	16	16	04	01	21	61	78
19	–„–	9	*57*	40	20	11	16	21	04	61	78
20	Algebra V	7	38	04	13	18	09	*54*	24	54	74
21	–„–	9	*42*	17	10	12	27	40	21	50	71
22	Algebra VIII	7	29	*37*	05	–08	–09	21	–08	29	54
23	–„–	9	*71*	02	13	03	07	26	03	60	78
24	Problems I, II	6	28	*68*	05	12	26	11	09	65	81
25	–„–	7	29	*68*	16	13	31	–06	11	71	84
26	–„–	9	34	*43*	07	14	41	04	32	60	78
27	Mark in Arithmetic	6	51	*54*	10	14	11	19	17	65	81
28	Mark in Algebra	7	*78*	28	03	13	09	21	24	82	91
29	–„–	8	*81*	30	10	03	19	07	07	81	90
30	–„–	9	*81*	26	02	06	21	04	02	77	88
	Eigenvalues per variable		.21	.12	.07	.06	.07	.04	.06	.63	

situations. We have combined the test variables of different situations in the same test battery. Then, we can analyse the structure of all tests in their common space.

The scope of this kind of analysis has been studied in the development analysis presented by Heinonen (1967). We use his technique without discussing the principles involved.

In this development analysis we have taken most of those ability and achievement tests which have been presented two or three times. For practical reasons we have omitted some tests, e.g. those algebra tests which have been presented at very different points of time in different groups. The variables of this analysis can be found in Table 3.19. The correlation matrix and the centroid factor matrix for these variables are in Appendix B (Tables 3.17 and 3.18). After the factorisation a Varimax rotation using 5 and 7 factors has been made. In my opinion, the number of factors should be 7, but we have tried also with a smaller number of factors to see if we could get the essential factors in this way. Both these rotated matrices are presented in Table 3.19 and 3.20.

Interpretation of the factors in Table 3.19:

Factor I. *Factor of school success in Algebra.* There are high loadings in the same variables as the same factors in Tables 3.7 and 3.10.

Factor II. *Problem solving factor.* There are high loadings in problem tests and the mark in Arithmetic.

Factor III. *Numerical factor.*

Factor IV. *Visual-reasoning factor.*

Factor V. *Factor of the test 'Form Boards'.*

Interpretation of the factors presented in Table 3.20:

Factor I. *Factor of school success in Algebra.*

Factor II. *Problem solving factor.*
Factor III. *Numerical factor.*
Factor IV. *Factor of the test 'Figures'.*
Factor V. *Factor of the test 'Form Boards'.*
Factor VI. *Factor of Grade 7 algebra.*
Factor VII. *Factor of the test 'Number Series'.*

The factor structure of these two matrices is for the first five factors almost equal. In both cases there are, firstly, a school success factor, a problem solving factor and a numerical factor. The others are factors with very high loadings for a single test. There is no common visual-reasoning factor though we have interpreted factor IV in Table 3.19 under this name.

We are not primarily interested now in the contents of the factors, but in the development of the loadings on the test variables in this space. We can also investigate here the development of the communalities, if there are no essential differences in the form of the distribution of scores in subsequent test situations. To get this development well defined, we use the space of seven factors.

We first give the differences between the loadings of the two presentations of the same test: loading in Grade 9 minus loading in Grade 7 (Grade 6 in tests 'Number Series', 'Arithmetic' and 'Addition'). These are given for each factor in Table 3.21. Then we have counted the sums of the squares of these differences (Σd^2) for each test and taken its square root. This is the length of the difference vector

$$e = \sqrt{\Sigma d^2}$$

and these are also given in Table 3.21 in this seven-factor space (e_7). Finally, there are the absolute values of the differences of the lengths of test vectors

$$|\triangle h| = |h_2 - h_1|.$$

Here the communalities are taken from Table 3.20.

The test 'Equations' was presented in Grade 6 after an experimental period and in Grade 7 after the summer holidays. During this period pupils had forgotten much of the equation solving technique, and the level of performance had decreased (c.f. Table 6.8). There has been a decrease in the numerical loading and an increase in the school success factor, problem factor and reasoning factor loadings in Table 3.19, which may be caused by these situational changes. The change in the factor structure from Grade 7 to Grade 9 has already been interpreted as the automatization of performance in equation solving. There is an essential decrease in the loadings of problem and reasoning factors, and an increase in the loading of the numerical factor. The test 'Equation' has the greatest difference vector in Table 3.21 though this is partly caused by the diminishing vector length.

The test 'Algebra VIII' was in Grade 7 mostly loaded in the problem solving factor but in Grade 9 in the factor of school success in Algebra. At the same time the length of the vector has increased essentially. This test variable still has in Grade 7 much variance which is perhaps connected with the situational factors. Besides,

Table 3.21. Differences between Two Testing Situations in the Seven Factor Space

Test	Factors I	II	III	IV	V	VI	VII	e_7	$\lvert\triangle h_7\rvert$
Number Series	09	20	11	00	24	−21	−10	.41	.02
Arithmetic	15	09	−10	−01	07	−34	−05	.40	.09
Addition I, II	−02	03	−01	04	15	01	−14	.21	.01
Figures	−10	00	07	01	02	09	−08	.17	.00
Form Boards	13	−01	−03	−07	03	−13	03	.20	.01
Equations	10	−34	22	−30	03	−34	01	.62	.14
Algebra I, II	−01	11	−02	08	10	−07	−36	.40	.07
Algebra III	−12	12	−04	11	10	02	−28	.36	.09
Algebra V	04	13	−03	−06	18	−14	−03	.27	.03
Algebra VIII	42	−35	08	11	16	05	11	.59	.24
Problems I, II	05	−25	09	01	10	10	21	.37	.06
Mark in Algebra	03	−02	−01	−07	12	−17	−22	.31	.03

the rapid understanding of principles used in algebra helps in this phase. In Grade 9 the differences have been caused by the training and this is common with the school success in Algebra on the whole. The increase in the length of test vector has partly affected the great value of the difference vector, but there is also essential change in the direction of the test vector.

The lengths of the difference vectors are rather great also in some other achievement tests, e.g. 'Algebra I, II'. There are not, however, changes in the factor loadings which could be interpreted well. This is in accordance with the result in the transformation analysis that the abnormal transformation is greater in the tests 'Equations' and 'Algebra VIII' than in other achievement tests. There are also some long difference vectors among the ability test variables (tests 'Number Series' and 'Arithmetic'), but these are obviously due to the small changes between factors.

We have presented in Table 3.18 the communalities of these variables for 12 factors. These are not much higher than the communalities in the seven factor space, but in any case, this causes only small discrepancies in the values of difference vectors and vector differences. There are also unsystematic variations in the factor structure which affect the differences in Table 3.21. We cannot, however, estimate the effects of these matters and the reliability of these difference loadings cannot be evaluated.

Summary and Discussion

We have obtained by means of factor analyses two ability factors, a visual-reasoning factor and a numerical factor. There is still a third factor, especially in Grade 9, which has been interpreted as a reasoning factor, but its constancy is not high and it is connected with school achievements as well. Numerical factors in Grade 7 and in Grade 9 have a good correspondence and they are well defined by the variables 'Addition I, II' and 'Multiplication'. Also the variable 'Work Test Addition' is highly loaded in this factor though its testing time was long (42 minutes). The same automatization of performances can be seen in many situations when the task is similar.

The visual-reasoning factor is not completely identical in Grade 7 and Grade 9. In both cases, this factor is determined by some reasoning and some visual tests. Thus, there are no separate reasoning and visual factors which is a natural result when this battery of tests is used. There are moderate changes in the location of the reasoning tests according to our development analysis. This may also decrease the constancy of the visual-reasoning factor. There is abnormal transformation in tests 'Form Boards' and 'Cubes' in Table 3.16. The location of the test 'Form Boards' has not changed in the same way in the development analysis. Then, the abnormal transformation may be affected by the changes of the reasoning factors.

The *Factor of complex Algebra* in Grade 7 corresponds best with the *Factor of school success in Algebra* in Grade 9 according to the transformation analysis. The variable 'Mark in Algebra' is still connected in Grade 7 with the simple algebraic tasks, but in Grade 9 with more complex algebraic tasks. This variable has kept its location rather well in the development analysis, but the variable 'Mark in Arithmetic' in Grade 6 is loaded mostly in the problem solving factor.

There are clear changes in the factor structure in the tests 'Equations' and 'Algebra VIII'. The location of the test 'Equations' has shifted from the reasoning factors to the numerical factor, and the location of the test 'Algebra VIII' from the problem solving factor to the factor of school success in Algebra. In both these cases the performances have been automatized because of the school teaching during the experimental period. These various changes in the factor structure are in accordance with the automatization theory of Werdelin (Malinen, 1961, p. 27). Automatization must take place also in those simplifications which are present in the tests 'Algebra I, II' and 'Algebra III'. These are, however, loaded in the factor of

school success in Algebra already in Grade 7. These simple performances were perhaps automatized already before the first test situation and their factor structure did not change after that. It is also possible to suppose that the performances in these tests should be automatized so that they are loaded more in the numerical factor. Those changes are not to be seen here. This may be caused partly by technical reasons, because there was a ceiling effect in the distribution of scores in the test 'Algebra I, II' in Grade 9. Perhaps these performances include so many rules that these cannot be automatized in the same way as the simplest additions.

We have used the symmetric transformation analysis to compare the factor structures between Grade 7 and Grade 9. This comparison has been made without the coefficients between the tests in different grades. In our situation we know the constancy coefficients of the tests. In those situations the rotation method presented by Werdelin (1962, pp. 196–204) can also be used. However, our method of checking the invariance of factor structures has also given suitable results.

We have checked the invariance of test structures by using the method of development analysis. The factors of the development analysis are not identical with those in the previous analyses. In this case we could use the factors in Table 3.19, but we have checked the results by using also the factors in Tables 3.7 and 3.10. The description of the changes has been completed by presenting the difference vectors and the vector differences in Table 3.21. We have estimated also the angles between the vectors of the same test variable in Grade 7 and Grade 9, but these did not give essentially new information.

Chapter 4. ATTITUDE TESTS AND THE STRUCTURE OF THE FIELDS OF ATTITUDES

Pupils' Attitude Tests

In our simplified model for mathematics learning there is a frame for affective domain variables. We have in the main measured attitude to and interest in Algebra. Besides, there are some ratings of the pupils' attitudes to school work in general. All of these are called attitude ratings. We have obtained information from pupils as well as from teachers, but the teacher questionnaire will not be presented until the third section. By means of these methods we have obtained only reports about opinions and attitudes. These measurements do not cover the whole affective domain, because we have no means of measuring the effects of, for example, the social situations in the school.

These instruments, their instructions and the distribution of the scores can be found in the Pedagogical Library of Alppilan yhteislyseo and in the Institute of Education, University of Helsinki. We give here only a short description of the instruments. The text, which is presented here in examples, is translated from Finnish.

Test No. 34. Questionnaire 1. 8 items concerning the attitude to school work as a whole. This is an adapted form of a pupil's questionnaire constructed by Werdelin (1960, pp. 46–48). Examples:

2) During the lessons do you think wholly of other matters than what is being presented?

5) Are you too lazy in your school work?

6) Have you too little interest in school work as compared with extra-curricular activities?

7) Do you spend too little time doing your homework so that your school achievement suffers from this?

8) Do you show too little ambition in your school work?

The pupils answered these questions using the scale: very often, often, average, seldom, very seldom. Items Nos. 2, 3, 5, 6, 7, and 8 were closely connected with the factor *Interest in School Work* in Werdelin's study (1960, pp. 108–110). The other items were included only to get more variation among the items.

We will form a variable for interest in school work, but we cannot be certain that the test functions as in the USA. The present writer has, therefore, made a cluster analysis of the structure of this questionnaire and published its results in the research report of Alppilan yhteislyseo (Malinen, 1968). It was stated that items Nos. 5, 7, 6, 8, 2, and 3 formed a cluster and we can combine them as a variable. However, we have omitted item No. 3, because it was the last in the cluster and it could not be included in the corresponding variable of teachers. We have formed a variable 'Interest in School Work' by adding the scores of items Nos. 2, 5, 6, 7, and 8. The highest intercorrelation between the items was .85, which is an estimate of the reliability of the items.

Test No. 35. Questionnaire 2. The pupils made paired comparisons of school subjects according to how they liked them. These subjects were Religion, Composition, History, Biology, Geography, Algebra, Geometry, Swedish, and English/German. We have formed a variable by counting the number of times Algebra was chosen. We call this variable 'Pleasantness of Algebra'.

The distribution of this variable was skew in Grade 7, because 22 per cent of the pupils did not choose Algebra in any situation. On the contrary, in Grade 9, 15 per cent of the pupils chose Algebra in all possible situations.

Test No. 36. Questionnaire 3. This includes 30 items about the attitudes to Algebra. All the items included are presented in Appendix B (Table 4.2). The pupils answered using the scale "strongly agree", "agree", "undecided", "disagree", "strongly disagree". These answers were given numerical values from 1 to 5. Many of these items were already in Werdelin's attitude inventory (1960, pp. 126–152; 1966 a). We have omitted the items which dealt with the attitude

to school work on the whole and to the teacher. Instead of that, there are some new items concerning attitudes to home exercises and the affective component of attitude. We have presented a factor analysis of this questionnaire in the second section of this chapter.

Test No. 37. Aspiration Level Test. This originates from the report by Worell (1959). The items:

1) How hard do you work in Algebra as compared to your classmates?

2) How do you think your success in Algebra is as compared with that of your classmates?

3) Let us assume that you are willing to continue in the higher grades in this school. How well do you expect to do in Algebra as compared to other pupils?

4) If you really tried to study Algebra with your whole capacity, how would you succeed as compared to your classmates according to your evaluation?

5) How well would you like to do in Algebra in order to be reasonably well satisfied according to your own standards?

The pupils answered using the nine-point scale, where the outmost frames were described as "more than the others" and "especially little" in item No. 1, and in the other items "especially well" and "especially little". The answers were scored 1 to 9. Then, four variables were formed using the differences of the scores:

A. *Capacity to Try More* (question 4 minus question 1). A high difference score indicates a great capacity, which the pupil has if he tries with his whole capacity.

B. *Gain in School Success* (3 minus 2). A high score indicates that the pupil estimates that he has a better performance than earlier.

C. *Capacity in School Success* (4 minus 2). A high score means that the pupil estimates that he has a greater capacity than the present school performance indicates.

D. *Dissatisfaction with Success* (5 minus 2). A high score means that the pupil is dissatisfied with his present performance in Algebra.

These four variables were almost normally distributed and the difference score values were from —3 to +7. Worell has used these variables to study the reality–irreality level of pupils and he stated that the pupils with good reality in their estimates succeeded in academic performance (Worell, 1959, p. 54). He then used these as bipolar variables. We have only few negative estimates and we do not get unrealistic estimates in the negative direction. Thus, we use them as unipolar variables. All these measure the aspiration level, but their factorial structure has not been studied by Worell.

We cannot be sure that the scales of these variables are interval scales, though the scores are normally distributed. This may be caused by many random errors and these variabels have low reliability. We can present here the same criticism which was presented already in Chapter 2 (p. 10) concerning the forming of difference scores. The greatest intercorrelation of these variables in our study is .58, which is rather low.

Test No. 38. Marks Estimate. Before the beginning of an ordinary written examination the pupils estimated their mark in this examination on a scale from 1 to 10. The examination was then marked using the same scale. The difference score (final mark minus estimated mark) is the variable 'Marks Estimate'. The scale of this variable was from —5 to +4 and the scores were almost normally distributed.

This variable measures the reality of the estimate, but also many random factors affect the variance. We can argue that the high scores with different signs have the same meaning as a measure of unreality. We have, however, taken negative values to mean that the pupils overestimate their own knowledge and positive values to mean that they underestimate it. The problem remains unsolved, because the reliability of this variable was only .20–.30 using the parallel test method. Thus, this test is of little value.

Tests Nos. 34, 35, and 36 were presented for the first time in Grade 7 in May 1962 and the retesting took place in Grade 9 in April 1964. The constancy coefficients between these two occasions for our variables are found in Table 4.19. Tests Nos. 37 and 38 were presented only in Grade 9, Spring 1964. Though there is defectiveness in the forming of the scales of these variables we assume that they have been measured by means of interval scales.

The Factor Structure of Questionnaire 3

In the preliminary treatment of Questionnaire 3 items Nos. 13, 17, 19, and 27 were excluded because of the skewness in the distributions of the scores. Item No. 24 was excluded, because it was identical with item No. 3. By presenting the same item twice we obtained an estimate of the item reliability. It was .93 in Grade 7 and .87 in Grade 9.

There remained 25 items for further analysis. Items Nos. 3, 6, 15, 20, and 22 were scored in the inverse direction to the others. Then, a factor analysis was made for these 25 variables. It is the writer's opinion that the structure is clearer in Grade 9. Thus, we have made a complete factor analysis in Grade 9, but only a preliminary analysis in Grade 7.

The correlation matrix and the centroid factor matrix for the variables in Grade 7 are in Appendix B (Tables 4.1 and 4.2). In the factor matrix there are also the statements of these items translated into English. We have taken five factors for further analysis. Then, we have made a preliminary rotation of factors using Ahmavaara's idea (Vahervuo & Ahmavaara, 1958, pp. 90–100). We have first formed the cosine matrix by dividing the product-moment correlations with the length of test variables according to the formula

$$\cos \varphi_{ij} = \frac{r_{ij}}{h_i \cdot h_j}.$$

The communalities in this formula have been estimated by means of the communalities in Table 4.2 for 8 factors (h_8^2). These cosine values are presented in Table 4.3. There are only those values which are greater than .80, because these are important in clustering. In the second column there are values of $\frac{|a|}{h}$, where a is the loading of each variable in the first factor and

Table 4.3. Cosine Matrix, Questionnaire 3, Grade 7

Factor	$\frac{\lvert a\rvert}{h}$		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
	.11	1																									
5/V	.96	2							80		94			93			96	83				92		90	88		
1/III	.57	3																									94
	.25	4																									
	.81	5																							94		
1/I	.07	6										91							92								
4/IV	.82	7		80									84	85							96			87	82		
1/II	.55	8																					83				
3/V	.90	9		94												92	97	83						97			
3/I	.20	10						91																			
1/IV	.74	11							84												94				85		
2/V	.89	12		93					85									92				86		89			
	.74	13																			95						
	.86	14									92						95				83			83			
	.90	15		96							97					95								87			
1/V	.84	16		83							83			92										81			
2/I	.36	17						92																			
	.45	18																									
2/IV	.88	19							96				94		95	83								81			
	.81	20		92										86													
2/II	.61	21								83																	
4/V	.96	22		90					87		97			89		83	87	81			81				90		
3/IV	.92	23		88			94		82				85											90			
	.47	24																									
2/III	.73	25			94																						

Decimal points have been omitted.

h the square root of the communality of each variable. Items Nos. 1, 4, 18, and 24 had no cosine values greater than .80, and we have omitted them because they do not form a cluster with other variables.

We take first the variable which is farthest away from the first centroid factor. This has the least $\frac{|a|}{h}$ value (variable 6). This has then been combined with the other variables, which are closely connected with it (variables 17 and 10—high cosine values). These form the first cluster (factor I). The following base vector is variable 8, which forms with variable No. 21 factor II, etc. The factor and the order of variable in this factor are in the first column of Table 4.3.

After four factors there remain many variables, which all lie closely together with the first centroid factor. We have taken variables Nos. 16, 12, 9, 22, and 2 in the last cluster, because they form a great unified set of variables and because this cluster corresponds better to factor IV in Grade 9 analysis.

Interpretation of these factors:

Factor I (variables 6, 17, 10). *Tendency to try to study Algebra.*

Factor II (variables 8, 21). *Dissatisfaction with results.*

Factor III (variables 3, 25). *Attitude to Algebra syllabus.*

Factor IV (variables 11, 19, 23, 7). *Attitude to success in Algebra.*

Factor V (variables 16, 12, 9, 22, 2). *Affective attitude to Algebra.*

Only a preliminary rotation was made in Grade 7, but in Grade 9 we have used the complete cosine rotation of Ahmavaara–Markkanen. The correlation matrix and the centroid factor matrix for the same variables in Grade 9 are in Appendix B (Tables 4.4 and 4.5). We have also taken five factors here. In the rotation, variable No. 1 was not accepted as a base vector because of its low communality. The best two solutions, according to the criterium presented in the rotation programme, were taken

Table 4.6. Cosine-rotated Factor Matrix, Questionnaire 3, Grade 9

No.	Items (shortened)	Factors I	II	III	IV	V
1	more practical topics	15	−18	*50*	21	−31
2	difficult subject	21	27	07	*56*	−15
3	more home exercises	*66*	00	00	00	00
4	not interesting subject	33	−13	25	*46*	−14
5	afraid of failing	−09	46	−13	*58*	04
6	tried the best to succeed	−16	−05	−04	25	*76*
7	depressed by poor achievement	00	*62*	00	00	00
8	not satisfied in relation to abilities	00	00	*61*	00	00
9	dull subject	30	−09	−06	*81*	04
10	could try more if needed	−15	−05	11	−16	*47*
11	do poorly in algebra	11	*46*	43	−04	04
12	teaching has progressed too fast	07	40	−08	*41*	14
13	not satisfied as compared to other subjects	−08	39	*51*	24	−19
14	gives a feeling of safety	43	−01	03	*52*	19
15	prefers other school work	00	00	00	*75*	00
16	too great demands in algebra	49	−07	01	*54*	10
17	done the best for home exercises	00	00	00	00	*80*
18	many unnecessary parts	*48*	−27	41	34	−13
19	satisfied with the results	15	*51*	26	18	22
20	difficult to understand principles	*46*	03	01	40	06
21	too lazy to study algebra	−02	12	05	28	[illegible]
22	makes irritable and restless	07	32	−15	*58*	23
23	more elementary exercises	17	*58*	16	13	19
24	only well-talented understand	07	07	−14	*49*	−13
25	more difficult exercises	*64*	13	−04	11	18

and in Table 4.6 we have presented the best solution.

Interpretation of factors (The first variable is the base vector.):

Factor I (variables 3, 25, 16, 18, 20). *Attitude to Algebra syllabus.*

Factor II (variables 7, 23, 19, 11). *Attitude to success in Algebra.*

Factor III (variables 8, 13, 1). *Dissatisfaction with results.*

Factor IV (variables 15, 9, 5, 22, 2, 16, 14). *Affective attitude to Algebra.*

Factor V (variables 17, 6, 21, 10). *Tendency to try to study Algebra.*

We have used the same names for the factors of these two analyses, though the factors did not include exactly the same items and we have not studied the correspondence of these factor structures. In the interpretation we have made use of Werdelin's results (1960 and 1966 a).

For further analysis we form the following new variables by adding the raw scores for test items:

The variable 'Dissatisfaction with Results' is in both cases not well defined. On the whole, the composite variables in Grade 9 are determined by more items than those in Grade 7. The variable 'Affective Attitude to Algebra' has been central in both analyses and it is closely connected with the first centroid factor both in Grade 7 and Grade 9 (Tables 4.2 and 4.5).

The definition of these variables in the field of items is not exactly known. According to the intercorrelations of factors in Table 4.7 there are relatively high correlations between the first and second variables and correspondingly between the third and fifth variables. These factors are not, however, indentical with the new variables.

Table 4.7. Intercorrelations of the Factors in Table 4.6

	Factors			
	I	II	III	IV
II	42			
III	−22	06		
IV	15	37	08	
V	−06	−04	42	01

	Items	
Variable	Grade 7	Grade 9
Attitude to Algebra Syllabus	3, 25	3, 18, 20, 25
Attitude to Success in Algebra	7, 11, 19, 23	7, 11, 12, 19, 23
Dissatisfaction with Results	8, 21	8, 13
Affective Attitude to Algebra	2, 9, 12, 16, 22	2, 5, 9, 14, 15, 16, 22
Tendency to Try to Study Algebra	6, 10, 17	6, 10, 17, 21

The Teacher Questionnaire

As in the monograph by Werdelin (1960), we have tried to get the same information from teachers as from pupils. Test No. 34, Questionnaire 1, is for pupils and after adaptation we have produced:

Test No. 39. Teacher Questionnaire, which includes 8 items, the same ones as in the pupils' questionnaire, but differently phrased and in a different order. Examples:

1) Is the pupil too lazy in your opinion?

3) Do you find that during the lesson the pupil thinks only of other matters than what is being presented?

7) Does the pupil spend too little time doing his home-work so that his school achievement suffers from this?

8) Does the pupil show too little ambition in school work?

The teachers were rated using the scale 1–5 on a rating form:

RATING FORM	Question No. __		Teacher: ______			
III A (class)	1	2	3	4	5	
Aarnipuu, Vuokko Alén, Marja-Leena etc.						

This rating form was given separately to each class. In Grade 7 the names of all the pupils who were studying in these classes were written. In Grade 9 the names of the students who had left at this time were removed but no new names were inserted. In the instruction the teachers were advised to use a normal distribution in the ratings for each class separately. The extreme columns in the rating form were described:

The pupil shows the behaviour indicated by the question to a very low (high) degree, as compared to his classmates.

This questionnaire was given in Grade 7 in January 1962 and in Grade 9 in February 1964 to the mathematics teacher, to the Finnish language teacher and to one foreign language teacher (Swedish, English or German). The items were scored on a scale from 1 to 5 as in the pupils' questionnaire.

The present writer has previously reported some results of this questionnaire (Malinen, 1968) and we will refer to them shortly. In spite of the instructions, some teachers had difficulties in filling up the rating form for questions 4, 5, and 6, and we have omitted these items in the further treatment of the data. There was skewness in the distributions of items 2 and 3 as the frequency of group 1 was greater than expected. However, it was possible to form item variables also from these items.

A cluster analysis was made of these variables. Items 1, 7, 8, and 3 made a cluster and only item 2 remained outside. The same result is reported also in Werdelin's study and he called this factor *Interest in Mathematics and School Work as Perceived by Teachers*. We make a combined variable from items 1, 3, 7, and 8 by adding the raw scores and we call this 'Teacher Rating'. The highest intercorrelations between the item variables (question 1 and 7) are .91–.92. The constancy coefficients for the combined variable between the two occasions (interval about two years) are .76–.89 for each class separately. Though the teachers had difficulties in making ratings in some questions, the ratings included in this variable are very constant.

Structure in Grade 7

In the affective domain we have eight variables, which have been measured both in Grade 7 and in Grade 9. These are:

No.	Variable	Test
1	Interest in School Work	Questionnaire 1
2	Pleasantness of Algebra	Questionnaire 2
3	Attitude to Algebra Syllabus	Questionnaire 3
4	Attitude to Success in Algebra	–,,–
5	Dissatisfaction with Results	–,,–
6	Affective Attitude to Algebra	–,,–
7	Tendency to Try to Study Algebra	–,,–
8	Teacher Rating	Teacher Questionnaire

We have changed the direction of some variables so that a high score in all cases means positive attitude. We have made a factor analysis of results in Grade 7. The correlation matrix and the centroid factor matrix are given in Appendix B (Tables 4.8 and 4.9). This factor matrix was then rotated using 3 factors, as there were only slight loadings after 3 factors. The rotation was made using the Varimax method and the result is in Table 4.10.

Interpretation of these factors:

Factor I. *Affective attitude to Algebra.*

Factor II. *Attitude to Algebra syllabus and other school work.*

Table 4.10. Varimax-rotated Factor Matrix, Attitude Variables, Grade 7

No.	Variables	Factors I	II	III	h^2
1	Interest in School Work	09	09	*72*	53
2	Pleasantness of Algebra	*76*	02	00	57
3	Attitude to Algebra Syllabus	45	*67*	21	70
4	Attitude to Success in Algebra	*77*	20	28	70
5	Dissatisfaction with Results	*37*	−14	35	28
6	Affective Attitude to Algebra	*76*	40	16	77
7	Tendency to Try to Study Algebra	13	52	*54*	59
8	Teacher Rating	01	*81*	−02	66

Factor III. *Interest in Algebra and other school work.*

Variables 1 and 8 are loaded in different factors, though they should measure the same interest in school work. They belong, however, to a different reference system and have, therefore, different dimensions. The communality of variable 5 is very small. We have already stated that this variable was not well defined because there were only two determining items. A discussion about the nature of these factors will be made after the comparison of factor structures.

Structure in Grade 9

We first made a factor analysis in Grade 9 for those variables which were tested also in Grade 7. The correlation matrix and the centroid factor matrix are in Appendix B (Tables 4.11 and 4.12). The rotation using three factors was also suitable here. The Varimax-rotation of this factor matrix is presented in Table 4.13.

Interpretation of these factors:

Factor I. *Affective attitude to Algebra.*

Factor II. *Interest in Algebra and other school work.*

Factor III. *Dissatisfaction with results in Algebra.*

In this matrix, teachers' ratings and pupils' ratings of the interest in school work are both loaded mainly in factor II, but the communality in the 'Teacher Rating' has been very much diminished as compared with the Grade 7 analysis. Factor I, *Affective attitude to Algebra*, is dominant in both analyses and is closely connected with the first centroid factor.

We have also made a factor analysis for all variables in the affective domain in Grade 9. This battery of variables includes the 'Mark in Algebra' as does the analogous battery in the cognitive domain. Besides, we have a new vari-

Table 4.13. Varimax-rotated Factor Matrix, Common Attitude Variables, Grade 9

No.	Variable	Factors I	II	III	h^2
1	Interest in School Work	−02	*70*	10	50
2	Pleasantness of Algebra	*76*	03	27	65
3	Attitude to Algebra Syllabus	*72*	08	−09	53
4	Attitude to Success in Algebra	*62*	17	43	60
5	Dissatisfaction with Results	16	23	*53*	36
6	Affective Attitude to Algebra	*82*	06	22	72
7	Tendency to Try to Study Algebra	02	*70*	13	51
8	Teacher Rating	19	*55*	12	35

able 'Average Mark'. This is the average of the marks given in Religion, Finnish, Biology, Geography, History, Swedish, German/English, Algebra, Geometry, and Physics. The same three teachers, who have rated the pupils, are dominant also in this variable, because they give together five marks and they distribute their marks considerably.

The analysis includes the following variables:

No.	Variable	Test (Time)
1–7	As before	
8	Capacity to Try More,	Aspiration Level Test
9	Gain in School Success	–„–
10	Capacity in School Success	–„–
11	Dissatisfaction with Success	–„–
12	Marks Estimate	Marks Estimate
13	Mark in Algebra	(May 1964)
14	Average Mark	(May 1964)

The correlation matrix and the centroid factor matrix are given in Appendix B (Tables 4.14 and 4.15). Five factors were used in the rotation, which was made according to the cosine-rotation system of Ahmavaara–Markkanen. The last two variables were passive in the rotation, because they do not belong to the attitude variables. We have taken the best two solutions according to the criterium presented in the rotation programme and the better of them is shown in Table 4.16.

Interpretation of factors:

Factor I. *Dissatisfaction with results in Algebra.*

Factor II. *Affective attitude to Algebra.*

Factor III. *Interest in Algebra and other school work.*

Factor IV. *Aspiration level of Algebra.*

Factor V. *Common school success.*

We have also made the rotation with four factors, using Varimax rotation. Then, variables 14 and 15 were excluded from the analysis. The rotated matrix is presented in Appendix B (Table 4.17). We got in this analysis four factors, which can be interpreted in the same way as the previous factors, I–IV. Only the common school success factor failed to appear. The variable 'Teacher Rating' did not get so high a communality here as in Table 4.15, where variables Nos. 14 and 15 were included in the first factorization. The rotation with 5 factors indicates that the variable 'Teacher Rating' is not essentially connected with the factors obtained from other attitude variables. These factors, I–IV in Table 4.16, are almost independent of the rotation system, and three of them (factors I–III) are almost identical with the three common attitude variable factors in Table 4.13. Only factor IV, *Aspiration level of Algebra*, is new.

Little was known about the factor structure of the test 'Aspiration Level' beforehand. The variables formed from this test have moderate communalities and variables 10 and 11 give new

Table 4.16. Cosine-rotated Factor Matrix, All Attitude Variables, Grade 9

		Factors				
No.	Variable	I	II	III	IV	V
1	Interest in School Work	27	61	*77*	–21	45
2	Pleasantness of Algebra	05	*70*	–09	10	–06
3	Attitude to Algebra Syllabus	–39	*42*	–26	32	27
4	Attitude to Success in Algebra	32	*44*	–10	17	–04
5	Dissatisfaction with Results	*63*	00	–00	00	00
6	Affective Attitude to Algebra	–00	*86*	00	–00	–00
7	Tendency to Try to Study Algebra	50	49	*86*	–13	18
8	Capacity to Try More	–00	–00	*71*	00	00
9	Gain in School Success	29	–28	–24	28	*30*
10	Capacity in School Success	–06	15	36	*65*	19
11	Dissatisfaction with Success	–00	00	–00	*76*	–00
12	Marks Estimate	02	–11	–09	08	*29*
13	Teacher Rating	00	–00	–00	–00	*92*
14	Mark in Algebra	–12	30	–22	22	*59*
15	Average Mark	–24	13	–09	13	*94*

information according to the subjective analysis of the present writer, which has not been obtained from other tests. We may assume that these variables indicate the differences in the organization of a value system, which is level 4 in Krathwohl's taxonomy (1964, pp. 154–164).

Attitude formation, as in factors II and III, is typical of level 3, Valuing, in this taxonomy. There is a factor for the affective attitude, which was measured in the same way in three tests. Thus, this is not a test-bound factor. The factor *Interest in Algebra and other school work* is a measure of the cognitive component of attitude. This is partly connected with the curriculum, but there is no special factor for interest in mathematics. Interest in school work indicates the pupil's beliefs about his behaviour in school learning situations. This is not the same as teachers' beliefs about the same matter. Teachers look upon the students' attitudes in the same way as they look upon school success, but pupils have an estimate which is connected with their personality. Werdelin (1966 c) has obtained corresponding results when investigating teacher ratings, peer ratings, and self ratings of behaviour in school.

There are also separate factors for affective component and cognitive component of attitude. These factors correlate strongly (coefficient .62 in Table 4.18), as suspected by Fishbein (1966, 203). There are different opinions about the hierarchy of these components (Fishbein, 1966, pp. 205–207; Karvonen, 1967, pp. 20–21). We cannot estimate here if the affective component is more stable than the cognitive, because our affective component is more precisely defined. Both components are found in Grade 7 and in Grade 9.

Table 4.18. Intercorrelations of Factors in Table 4.16

	Factors			
	I	II	III	IV
II	39			
III	40	62		
IV	49	46	14	
V	52	18	10	43

There is one factor, which we have interpreted as *Dissatisfaction with results in Algebra* (factor III in Table 4.13 and factor I in Table 4.16). It was not found in the analysis of Grade 7. It can perhaps be interpreted as a special attitude to Algebra. The variable 'Dissatisfaction with Results' was, however, weakly defined, and its communality is low. This may also be a factor without a stabile structure.

Comparison between the Factor Structures

We have eight variables, which have been tested both in Grade 7 and in Grade 9. The difference between testings was almost two years. The constancy coefficients between these two test situations are found in Table 4.19.

Table 4.19. Constancy Coefficients of Attitude Variables

No.	Variable	Coefficient (N = 119)
1	Interest in School Work	.74
2	Pleasantness of Algebra	.53
3	Attitude to Algebra Syllabus	.39
4	Attitude to Success in Algebra	.44
5	Dissatisfaction with Results	.29
6	Affective Attitude to Algebra	.55
7	Tendency to Try to Study Algebra	.34
8	Teacher Rating	.77

There is a high constancy coefficient in the variable 'Teacher Rating'. There is no great change in the variable 'Average Mark', either, as its constancy coefficient between Grades 7 and 9 is .84. There are high constancy coefficients also in other studies for the average mark (Sipinen, 1967, pp. 28–33 and 65–67). For pupil variables the highest constancy was in the variable 'Interest in School Work'. This may be caused by a higher reliability, but maybe pupils have a considerable stable cognitive component of attitude.

Our common variables were so few that we cannot estimate the constancy of factors in these frames very well. We have made the transformation analysis as in the cognitive domain,

but using the centroid factors in Tables 4.9 and 4.12. The transformation matrix L_{12} is in Table 4.20. There is a good correspondence between these factors which can already be interpreted almost like the rotated factors.

We see the amount of abnormal transformation for variables in Table 4.21. The coefficient for the residual is independent of the rotation. There isa great residual in the variable 'Teacher Rating'. This has a good constancy, but it has changed its factor loadings essentially in this three-factor system which is a reference system of pupils. The changes in variable No. 7 can be interpreted, as the tendencies to study algebra will usually change in Grade 9. There are different needs among those who leave school and among those who remain there.

Table 4.20. Transformation Matrix L_{12}, Attitude Factors

		Grade 7 factors		
		I′	II′	III′
	I′	0.955	−0.291	0.060
Grade 9	II′	−0.045	0.057	0.997
factors	III′	0.293	0.955	−0.041

Table 4.21. Residuals of the Variables in Transformation

No.	Variable	Residual
1	Interest in School Work	0.022
2	Pleasantness of Algebra	0.060
3	Attitude to Algebra Syllabus	0.046
4	Attitude to Success in Algebra	0.021
5	Dissatisfaction with Results	0.026
6	Affective Attitude to Algebra	0.009
7	Tendency to Try to Study Algebra	0.320
8	Teacher Rating	0.875

Summary and Discussion

We have analysed the structure of the fields of attitudes in different phases. Firstly, we have analysed the structure of item variables in the tests Questionnaire 1, Questionnaire 3, and Teacher Quetionnaire separately. In this way we have obtained variables from all attitude tests. Secondly, we have analysed the structure of these variables separately in Grade 7 and in Grade 9. Thirdly, a comparison between these structures was made by using transformation analysis.

Different factors for the affective and the cognitive component of attitude have been found in earlier investigations. We have found the corresponding factors both in Grade 7 and in Grade 9. These have been interpreted as *Affective attitude to Algebra* (affective component) and *Interest in Algebra and other school work* (cognitive component). There was a third factor both in Grade 7 and in Grade 9, but this could not be interpreted in the same way in both cases. The variable 'Teacher Rating' did not fit in well in this factor system and it had a great abnormal transformation in transformation analysis.

A wider analysis was also made in Grade 9. Then, a new factor, *Aspiration level of Algebra*, was revealed. According to the interpretation this is connected with the organisation of a value system. In the taxonomy edited by Krathwohl (1964) this factor is connected with a higher level of aims than the factors concerning the cognitive and affective component of algebra.

In this wider analysis a *Common school success* factor was also presented. This is a teacher factor which was determined by the variable 'Teacher Rating'. The variable 'Average Mark' was highly loaded in this factor and it seems that these two teacher ratings have basically the same content, because in all situations the teachers are rating the same behaviour that is valuable according to their reference system.

This analysis of attitudes is defective in many respects. The stability of factors was investigated using only the transformation analysis and the determining of factors was not made precise enough. However, these results correspond with the preliminary surveys concerning these attitudes. We have measured external impressions from pupils and from teachers concerning affective behaviour, and thus it is natural to get different results from pupils and from teachers, as is the case with Questionnaire 1 and the Teacher Questionnaire.

Chapter 5. TOTAL ANALYSIS OF PUPILS' CAPABILITIES

We have formed a battery of variables including the most important ones of those which were included in both the Grade 7 and the Grade 9 study. These variables are known from the analyses of previous chapters. We call this the total battery of variables. Because we have not included all variables which were in the previous analyses we cannot expect to obtain the same factors. It is our aim to check if the variables in the field of achievements and abilities are loaded in the same factors as the variables in the field of attitudes.

Structure of the Total Battery in Grade 7

The variables in this battery are given in Table 5.3. Variables Nos. 6–10 were obtained in Grade 6, all the others in Grade 7. The scores for the test 'Algebra IX' are in class B among the Grade 8 tests. The marks for Grade 7 were given in May.

The correlation matrix and the centroid factor matrix for this battery of variables are in Appendix B (Tables 5.1 and 5.2). After extracting four factors we have obtained the essential information from most variables and after six factors the loadings of all variables are already slight. We have rotated this matrix using both four and six factors. For our purpose the rotation with six factors is more important and it is presented in Table 5.3 using the Varimax rotation.

Interpretation of factors:

Factor I. *Observed behaviour indicating interest in school work.*

Factor II. *Reasoning and Algebra factor.*

Factor III. *Numerical factor.*

Table 5.3. Varimax-rotated Factor Matrix, Total Analysis, Grade 7

		Factors						
No.	Variable	I	II	III	IV	V	VI	h^2
1	Interest in School Work	*61*	−19	02	18	06	06	44
2	Pleasantness of Algebra	09	05	12	*82*	04	02	70
3	Attitude to Success in Algebra	20	27	05	*66*	22	26	67
4	Affective Attitude to Algebra	33	12	10	*77*	09	11	75
5	Teacher Rating	*82*	19	07	19	16	11	78
6	Number Series	14	35	03	12	08	*51*	42
7	Syllogisms	−01	*67*	−05	05	15	17	51
8	Arithmetic	−06	*48*	09	00	47	30	55
9	Addition I, II	01	16	*81*	09	07	11	71
10	Multiplication	17	02	*81*	13	05	02	70
11	Figures	10	13	07	07	*49*	31	37
12	Form Boards	13	09	01	08	*62*	01	42
13	Cubes	05	*51*	15	13	21	13	36
14	Equations	17	*48*	18	18	41	31	59
15	Algebra I, II	36	*50*	24	40	07	27	68
16	Algebra III, IV	32	*59*	22	42	10	21	73
17	Algebra V	28	18	14	20	17	*50*	45
18	Algebra IX	*48*	44	11	37	−01	01	57
19	Problems I, II	26	47	08	14	*52*	−11	59
20	Mark in Algebra	*52*	38	10	50	24	28	82
21	Average Mark	*74*	28	14	12	24	24	78
Eigenvalues per Variable		.13	.13	.08	.13	.08	.06	.60

Factor IV. *Affective attitude to Algebra and specific school success in Algebra.*

Factor V. *Problem solving factor.*

Factor VI. *Inductive reasoning and Algebra factor.*

When reducing the number of factors from six to four, the reasoning and problem solving factors have combined together. Besides, the factor *Affective attitude to Algebra and specific school success in Algebra* has higher loadings in the algebra tests and in the variable 'Mark in Algebra'.

The variable 'Interest in School Work' could not alone form the factor of the cognitive component of attitude. It is connected closely with teachers' observations, but has a rather low communality in this table. Other attitude variables form their own factor where many algebra tests have moderate loadings. The attitude variables are not loaded in the same factors as the ability variables. Factor III, *Numerical ability* is specific in nature. Except for tests Nos. 9 and 10 there are only slight loadings in the simple algebra tests. The reasoning and visual variables are loaded in the same factors as the achievement tests as in the previous analysis in Grade 7 (Table 3.7). Thus, in no case attitude and cognitive domain variable combine to form a factor. The variable 'Mark in Algebra' is loaded in most factors, but mostly in factors I and IV which have to do with the attitude variables.

Structure of the Total Battery in Grade 9

More tests were presented in Grade 9 than in Grade 7. In the total battery in Grade 9 we have also used more variables. Some of these new variables are the results of learning algebra and some are connected with the understanding of algebra. These variables are presented in Table

Table 5.6. Varimax-rotated Factor Matrix, Total Analysis, Grade 9

No.	Variable	Factors						h^2
		I	II	III	IV	V	VI	
1	Interest in School Work	*49*	02	08	15	−11	−00	28
2	Pleasantness of Algebra	02	20	*80*	03	13	12	71
3	Attitude to Success in Algebra	15	14	*66*	21	16	16	56
4	Affective Attitude to Algebra	10	10	*82*	11	−04	06	71
5	Teacher Rating	*84*	11	07	−06	22	21	82
6	Number Series	−09	*53*	21	17	25	34	53
7	Syllogisms	−09	*54*	26	17	10	46	61
8	Arithmetic	09	40	13	04	26	*48*	49
9	Addition I, II	−03	12	11	*73*	14	05	58
10	Multiplication	18	15	12	*71*	−05	−04	57
11	Figures	07	*60*	−04	15	06	10	41
12	Form Boards	14	*50*	15	−03	40	00	45
13	Cubes	09	*58*	21	07	−08	17	43
14	Equations	06	09	23	*48*	36	19	46
15	Algebra I, II	27	31	38	14	*43*	21	56
16	Algebra V	16	23	10	16	*50*	35	48
17	Algebra VIII	36	06	*52*	15	30	20	55
18	Algebra X	*47*	10	42	15	42	29	68
19	Algebra XI	28	16	43	26	*44*	16	57
20	Problems I, II	16	*53*	19	12	29	30	53
21	Comparisons	05	39	22	−01	14	*61*	60
22	Operations	19	20	12	16	09	*55*	42
23	Understanding	26	07	06	−07	06	*54*	37
24	Mark in Algebra	51	18	*55*	09	39	27	83
25	Average Mark	*81*	08	18	01	32	25	86
Eigenvalues per Variable		.11	.10	.13	.07	.07	.09	.56

5.6. The correlation matrix and the centroid factor matrix for these variables are in Appendix B (Tables 5.4 and 5.5). It was found here, too, that the loadings were slight after six factors. Thus, the rotated factor matrix in Table 5.6 is presented using six factors. Interpretation of factors in this table:

Factor I. *Observed behaviour indicating interest in school work.*

Factor II. *Reasoning factor.*

Factor III. *Affective attitude to Algebra and specific school success in Algebra.*

Factor IV. *Numerical factor.*

Factor V. *Second specific Algebra factor.*

Factor VI. *Factor of understanding mathematical principles.*

We have made a corresponding analysis with four factors, too (Table 6.2 in Appendix B). Then, the second specific algebra factor and the factor of understanding mathematical principles have combined together with the reasoning factor and with the factor *Observed behaviour indicating interest in school work.*

Factor I is connected with the teachers' observations as in Grade 7. Many algebra tests and 'Mark in Algebra' are connected with this factor. Factor II is almost a pure reasoning factor. The algebra variables are not connected essentially with this factor. Instead of that, algebra variables are connected with the affective attitude in factor II. as in Grade 7. The other factors are in the cognitive domain. Thus, no joining of attitude variables and ability variables is to be seen. The loadings of achievement variables have been divided in many factors, but the conclusions concerning these connections can be made only after more analysis.

Forming Intervening and Dependent Variables

In Chapter 2 we have analysed the tasks in algebra and then formed algebra tests which should measure these tasks. Besides, we have measured the abilities and attitudes of pupils. We have formed variables from the test scores making no distinctions between intervening and dependent variables and we have called them all information output variables. There are a few independent variables in this study and these will be treated only in Chapter 6.

It is now our purpose to form intervening and dependent variables for further analyses. This will be made in the main by using the analyses presented in the previous chapters. Mostly we have not interpreted the factors when we have presented the results of the factor analyses in these chapters, but only given the names of the factors. Now we will analyse further the content of those factors which form a basis for our new variables. When forming these new variables we cannot use the factor scores from the previous analyses because these factors are not "pure" enough for further analysis. The forming of intervening and dependent variables must be made by combining several results of the analyses in Chapters 3, 4, and 5.

Reasoning Ability

There are some factors, which are connected clearly with pupils' abilities, and we can form intervening variables on the basis of such factors. It is possible to form only one stable reasoning variable from reasoning and visual tests. It is perhaps in its purest form in the total analysis, Grade 9 (Table 5.6, factor II). This factor is present in slightly different forms in all analyses of the cognitive domain (factor III in Table 3.7, factor I in Table 3.10, factor II in Table 3.13, and factor II in Table 5.3) and it has shown good constancy in transformation analyses. We omit from this variable all tests which are connected with teaching. The test 'Form Boards' has also been excluded because of its low loadings. For the purpose of the present study we should properly give the individual tests the weights that maximize the connection between the combined variable and the factor. However, we have used the simple weight given below, which gives almost the same importance to all included variables, i.e. all variables have almost the same distribution. We have used the same weights in Grade 7 (Grade 6) and Grade 9.

We have then added the scores of the variables 'Number Series' (double score), 'Syllogisms', 'Arithmetic' (double score), 'Figures', and 'Cubes', and we call this new combined variable 'Reasoning Ability'.

The variable 'Reasoning Ability' is connected closely with the *General Reasoning Factor* and the *Deductive Factor* presented by Werdelin (1958; 1960). He has later analysed this reasoning factor more (Werdelin, 1966 b and d). The materialistic base of 'Reasoning Ability' is wide, including also visual tests, and it includes most elements typical of the general factor (Spearman's g factor) except the verbal tests. All tests which determine this variable include abstract thinking and some tests have still been too difficult for some pupils in Grade 7. Thus, this combined variable is better determined in Grade 9. Its field of definition is also wider in Grade 9, because in Grade 7 there was still a separate factor which was partly interpreted as an inductive factor (factor I in Table 3.7 and factor VI in Table 5.3).

The loadings of these variables which have been included in the 'Reasoning Ability' variable have not changed essentially in the development analysis made in Chapter 3. Thus, this variable is connected with the crystallized ability to solve abstract mathematical problems where the solving system must be discovered. It includes both inductive reasoning when the pupil tries to generalize his simple solving principles, and deductive reasoning when the pupil tries to apply his more general principles.

Numerical Ability

The numerical factor appeared in all analyses of the cognitive domain and its constancy in transformation analysis was considerable. It is easy to form a combined variable which will measure numerical ability. We have also here used simple weights both in Grade 6 and in Grade 9 and added the scores of the test variables 'Addition I', 'Addition II' and 'Multiplication'. This new combined variable is called 'Numerical Ability'.

The content of the variable 'Numerical Ability' is well known from the wide analysis made by Werdelin (1958). This variable measures automatization of well-established associations, which originally involved some form of reasoning. As presented in Chapter 3 this automatization occurs in the solving of simple equations but not in the simplifying of the expressions of algebra. The length of the performance does not change its factor structure if the task is simple (c.f. page 28). Numerical ability is specific in nature, most mathematical performances are not connected with this variable.

Attitude to Algebra

In the field of attitudes there appeared most clearly a factor which was interpreted in several analyses as the affective attitude to Algebra. It was not connected with any specific testing situation, because there were high loadings in this factor in variables from different tests. We form a combined variable by adding the scores of the variables 'Pleasantness of Algebra' (double score), 'Attitude to Success in Algebra', and 'Affective Attitude to Algebra'. The scores in Grade 7 and in Grade 9 do not correspond exactly to each other, because there were more items in Grade 9. This new variable is connected with the affective component of attitude, but we call it simply 'Attitude to Algebra'.

The combination of the variables presented above is more complex than the combination of performance variables, because the variable 'Pleasantness of Algebra' is measured in a different way as compared with the other variables. Besides, the variables 'Attitude to Success in Algebra' and 'Affective Attitude to Algebra' were closely connected with different factors in the item analysis of Questionnaire 3. Thus, in a more precise analysis we will use these variables separately and the combined variable 'Attitude to Algebra' only in a rough description of attitudes.

All the three variables mentioned before indicate some kind of self-rating of pupils' attitudes to Algebra. The role of those ratings in the measurement of the field of attitudes has been discussed by Werdelin (1968 b). These self-ratings have been closely connected with the school situations and they are pupils' impressions of how they regard the studying of algebra. Then, the school success in Algebra has a great effect on the atmosphere, but the converse may also be true, with the atmosphere

affecting the eagerness to work with algebra. The affective attitude to Algebra has retained its structure well when comparing the results of Grade 7 with the results of Grade 9. We can take it as an intervening variable for new processes where the previous attitude will affect the results.

Common School Success

In the field of attitudes there was a factor which was interpreted as *Interest in Algebra and other school work* (Tables 4.10, 4.13, and 4.16). This represented the cognitive component of attitudes. In Tables 5.3 and 5.6 there is correspondingly a factor *Observed behaviour indicating interest in school work*. We have the ratings from pupils and from teachers concerning the same questions but these are not connected closely with the same factor. Teachers' ratings are connected closely with the variable 'Average Mark'. The measurement of the cognitive component of attitudes is here so imperfect that we do not form any combined variable from this domain. We have got from teachers information about the observed behaviour and this does not differ much if the behaviour is measured using the Teacher Questionnaire or using the marks. The variable 'Average Mark' has a good constancy and for some purposes we use it as an indicator of 'Common School Success'.

There is also other evidence that the ratings made by teachers agree well with the marks they have given to the pupils (Werdelin 1968 b). We do not discuss here the content of the variable 'Common School Success'. It is an observed result of school learning processes and in this meaning a dependent variable. There are, however, situations where we can take it as an intervening variable. When the pupils are learning a new topic and we investigate the process during a short period, one prerequisite is common interest in school work and the habit of doing it. This is measured by the variable 'Common School Success' and this is an intervening variable for this new phase.

Understanding

In Tables 3.13 and 5.6 there was a factor which was interpreted as *Factor of understanding mathematical principles*. There were high loadings for the tests 'Comparisons', 'Operations', and 'Understanding'. These tests were given only in Grade 9, and we cannot estimate the constancy of this factor. According to the theoretical discussion in Chapter 2 the levels of understanding were important for the description of learning. Therefore, we will investigate more this domain of behaviour and form a new variable by adding the scores of the tests 'Comparisons', 'Operations', and 'Understanding' (double score). We call this combined variable 'Understanding'.

The tests which determine the combined variable 'Understanding' include reasoning in new situations ('Comparisons' and 'Operations') or understanding mathematical principles in abstract situations. In all cases, this means working at a high level of understanding. We have described the aims of mathematics teaching using Bloom's taxonomy (1956). According to the present writer's opinion, this combined variable measures working at the level of analysis and synthesis. The connection between the variables 'Reasoning Ability' and 'Understanding' is not clear. The factor of understanding mathematical principles had moderate loading also among the reasoning variables. It may be that if the tests 'Comparisons' and 'Operations' were practised a little more, they would be connected closely with the reasoning ability. Then, 'Understanding' measures high level performances which are needed at the beginning, but later on these tasks can be performed using the more crystallized (automatised) processes which have been measured by the variable 'Reasoning Ability'.

We can assume that the highest processes are the end products of learning. Thus, the combined variable 'Understanding' measures the last result we will get in the learning process. That is why we can regard it as a dependent variable. This variable is not, however, wholly connected with the aims of the teaching of algebra. We cannot accept beforehand that all pupils must reach this level in understanding when teaching them algebra. There is a lack of theoretical discussion concerning the aims of this area. In every case, we can accept the variable 'Understanding' as an intervening variable

when investigating the processes concerning the learning of algebra.

Simple Algebra

We have in our test battery many tests of simple algebra, and factors have been isolated in which these tests have been highly loaded (factor I in Table 3.7, factor III in Table 3.10, factor I in Table 3.19 and 3.20, and factor V in Table 5.6). In most cases the variables 'Algebra I, II' and 'Algebra III, IV' are connected with this factor, and in some cases also 'Algebra V'. We consider it important to form a variable of simple algebra, where there is also knowledge of the principles of simplifications. Thus, we have added the scores of the variables 'Algebra I', 'Algebra II', and 'Algebra V' (double score). We call this new variable 'Simple Algebra'.

The variable 'Simple Algebra' is a result of learning, which has been developed in school situations. There have been changes in factor structure in the variables 'Algebra I, II' and 'Algebra V' in Tables 3.7 and 3.10, but the loadings of these variables remain almost unchanged in the development analysis. The only essential change may be that these variables are connected closely with the variable 'Mark in Algebra' in Grade 7 but not in Grade 9. The combined variable 'Simple Algebra' is a dependent variable when we regard the first phases of the teaching of algebra. Later this variable indicates differences between pupils concerning the previous knowledge. Then, it is an intervening variable.

Complex Algebra

There are different topics of algebra in each grade and we can form new variables to measure the latest knowledge in algebra. In Grade 8 we have used the variable 'Algebra VI' in a specific comparison of the experimental groups (c.f. Chapter 6). The last topics of algebra in Grade 9 have been measured by a set of algebra tests. We have combined three complex tests in algebra, 'Algebra IX', 'Algebra XI', and 'Algebra XII' (double score) for a new variable 'Complex Algebra'.

This combined variable measures skills in working with complex material in algebra. In Table 3.13 the three variables presented before were loaded in the same factor, which was closely connected with the variable 'Mark in Algebra'. We have taken the variable 'Complex Algebra' as a dependent variable besides the variable 'Mark in Algebra', because the former is a pure performance variable.

Mark in Algebra

The most natural dependent variable in our investigation is the variable 'Mark in Algebra'. We have examined its connections with the performance variables and with the attitude variables in previous analyses, but its content is not yet widely discussed. The mark in Algebra is not given according to some mechanical system on the basis of the written examinations. According to the subjective analysis of the present writer the mathematics teacher proceeds in the following order: He examines the previous mark in Algebra. He looks at the row of numbers which the pupil has got from written examinations. He evaluates the mean value of the examinations. He ascertains whether the results have caused changes in the mark in Algebra. Thus, the new mark is not independent of the previous mark and this variable has a good constancy.

This method of marking means that the variable 'Mark in Algebra' is not a pure performance variable. A pupil's interest in discussing during the lessons affects this variable and it is not entirely connected with the latest topics of algebra. Such a delay has been established in the studies of Dahllöf (1967, pp. 207–209). This cannot easily be established in our study, because the simple and complex algebra tests are on the whole loaded in the same factors in Grade 9.

Summary

In the field of attitudes it is impossible to find a variable which would be suitable for a dependent variable. In Table 4.16 we have presented a factor interpreted as *Aspiration level of Algebra*. According to the present writer's opinion this measures a higher level of affective domain behaviour than the variable 'Attitude to Algebra'. Thus, this would be best suited as a

Table 5.7. Information about Intervening and Dependent Variables

Variable	Grade 7			Grade 9		
	Median	Mean	S.D.	Median	Mean	S.D.
Reasoning Ability	54.9	56.3	15.6	65.3	63.8	16.3
Numerical Ability	53.3	53.4	11.2	46.2	47.7	14.2
Attitude to Algebra	64.2	64.8	11.7	50.7	51.6	13.9
Common School Success	77.7	78.8	7.6	75.9	76.6	9.3
Understanding	—	—	—	23.1	23.5	7.1
Simple Algebra	43.3	43.7	9.4	47.3	47.0	9.1
Complex Algebra	—	—	—	23.5	24.1	9.6
Mark in Algebra	6.9	7.4	1.4	6.7	7.1	1.5

dependent variable. This factor is not, however, precisely determined by the two variables and its content in our investigation has not been clarified enough. Thus, we omit this factor in the further analysis. We omit also the other factors which have been found in some single factor analysis.

So, we have obtained a system of intervening and dependent variables. The division into these groups of variables is not universal but must be made according to the phase of learning. The medians, means, and standard deviations of these new variables are given in Table 5.7. The distributions of the scores can be seen in the Pedagogical Library of Alppilan yhteislyseo and in the Institute of Education, University of Helsinki. The scores have almost normal distributions, and we suppose that these variables have been measured on an interval scale.

The correlation coefficients between these variables (except 'Complex Algebra' and 'Mark in Algebra') are in Tables 5.8 and 5.9.

Table 5.8. Correlation Coefficients between Variables, Grade 7

	1	2	3	4
1 Reasoning Ability				
2 Numerical Ability	0.15			
3 Attitude to Algebra	0.32	0.25		
4 Common School Success	0.44	0.27	0.40	
5 Simple Algebra	0.55	0.35	0.53	0.57

Table 5.9. Correlation Coefficients between Variables, Grade 9

	1	2	3	4	5
1 Reasoning Ability					
2 Numerical Ability	0.28				
3 Attitude to Algebra	0.39	0.26			
4 Common School Success	0.25	0.14	0.30		
5 Understanding	0.59	0.35	0.37	0.59	
6 Simple Algebra	0.56	0.32	0.42	0.53	0.61

Tasks for Intervening Variables

Let us suppose there is a linear prediction system, where y is the dependent variable and x_1, $x_2, \ldots, x_6$ the independent variables. The corresponding standard scores are $z_y, z_1, z_2, \ldots, z_6$. Then, the prediction equation can be presented

$$z_y = \beta_1 z_1 + \beta_2 z_2 + \cdots + \beta_6 z_6$$

(McNemar, 1955, p. 178). We get the multiple correlation coefficient between the dependent variable and the independent variables from

$$r_{y\cdot 12\ldots 6} = \sqrt{\beta_1 \cdot r_{y\cdot 1} + \beta_2 \cdot r_{y\cdot 2} + \cdots + \beta_6 \cdot r_{y\cdot 6}},$$

where $r_{y\cdot n}$ ($n = 1, \ldots, 6$) is the correlation coefficient between the dependent variable and each independent variable (McNemar, 1955, p. 180).

In our situation we take as independent variables six intervening variables in Grade 7 and seven variables in Grade 9. There are two ability variables 'Reasoning Ability' and 'Numerical

Ability', three attitude variables 'Pleasantness of Algebra', 'Attitude to Success in Algebra', and 'Affective Attitude to Algebra', and one achievement variable 'Simple Algebra'. Besides, in Grade 9 we still have the combined variable 'Understanding'. We have not combined here the attitude variables because we want a more precise description of their role in prediction.

As dependent variables we have in Grade 7 the 'Mark in Algebra' and in Grade 9 the 'Mark in Algebra' and 'Complex Algebra'.

We have tested the linearity of the regression of the variable 'Mark in Algebra' (McNemar, 1955, pp. 268–272). There was no nonlinear regression between independent variables and this dependent variable. Thus, we meet no technical problems when we use this linear regression model. Regression analyses were made to determine the beta-coefficients and the multiple correlation coefficients. The significance of thedeviation from zero of the beta coefficients and the multiple correlation coefficients was tested by means of the t-test. These calculations were made for the whole experimental group.

The results in Grade 7 are in Table 5.10. The highest correlation coefficients were for the variables 'Attitude to Success in Algebra' and 'Simple Algebra'. It is natural that they are closely related to the 'Mark in Algebra', as they have interactions with the dependent variable. Success (high marks) affects positive attitude and *vice versa*. A high mark in Algebra during the previous periods helps in getting a high score in 'Simple Algebra' and conversely. These two independent variables include much of the variance which can be used when predicting the variable 'Mark in Algebra'. It may be that 'Attitude to Success in Algebra' and 'Simple Algebra' are the results of a pupil's reasoning ability and understanding of algebra. For this reason the beta-coefficient of the 'Reasoning Ability' is not very high.

The significance of the deviation from zero of the correlation coefficients $r_{y.n}$ in Tables 5.10 and 5.11:

	$r_{y.n} < 0.18$	$0.05 < p$
$0.18 <$	,, < 0.24	$0.01 <$,, < 0.05
$0.24 <$	,, < 0.31	$0.001 <$,, < 0.01
$0.31 <$	,,	,, < 0.001

Table 5.10. Regression Analysis, Grade 7

Dependent Variable	Mark in Algebra		
Independent Variables	$r_{y.n}$	Beta	Significance for Beta
Reasoning Ability	0.54	0.15	$p < 0.05$
Numerical Ability	0.26	−0.00	> 0.05
Pleasantness of Algebra	0.51	0.10	> 0.05
Attitude to Success in Algebra	0.73	0.34	< 0.001
Affective Attitude to Algebra	0.65	0.13	> 0.05
Simple Algebra	0.71	0.40	< 0.001
Multiple Correlation	0.84	$p < 0.001$	

Table 5.11. Regression Analysis, Grade 9

Dependent Variables	Mark in Algebra			Complex Algebra		
Independent Variables	$r_{y.n}$	Beta	Significance for Beta	$r_{y.n}$	Beta	Significance for Beta
Reasoning Ability	0.46	−0.03	$p > 0.05$	0.44	−0.12	$p > 0.05$
Numerical Ability	0.27	0.02	> 0.05	0.25	0.01	> 0.05
Pleasantness of Algebra	0.59	0.20	< 0.05	0.59	0.28	< 0.01
Attitude to Success in Algebra	0.59	0.17	< 0.05	0.59	0.20	< 0.01
Affective Attitude to Algebra	0.53	0.13	> 0.05	0.49	0.01	> 0.05
Simple Algebra	0.66	0.39	< 0.001	0.68	0.41	< 0.001
Understanding	0.49	0.16	< 0.05	0.53	0.23	< 0.01
Multiple Correlation	0.78		$p < 0.001$	0.80		$p < 0\ 001$

The results in Grade 9 are in Table 5.11. There are no essential differences between the results of the variables 'Mark in Algebra' and 'Complex Algebra'. In both cases the beta-coefficient of the 'Reasoning Ability' is negative but the coefficient does not differ significantly from zero. There are, however, significantly positive correlation coefficients between this and the dependent variables. Thus, reasoning ability has its effects in the attitude and achievement variables which have been developed during the learning situations. The variable 'Understanding' still has a moderate beta-coefficient. It affects the results of 'Complex Algebra' in a way which cannot be predicted by the other variables. The role of 'Simple Algebra' is still as important as in Grade 7, but the 'Attitude to Success in Algebra' has now lower coefficients.

The multiple correlation coefficients are rather high both in Grade 7 and in Grade 9 even if we have among the independent variables only variables concerning the learning of algebra directly. It may be that one part of the variance will be predicted by means of personality variables connected with the common school success, but the low reliability of all variables also prevents a complete prediction.

In this linear regression system pupils can obtain the same value in dependent variables by different combinations of independent variables. One pupil may be low in 'Pleasantness of Algebra' and in 'Understanding' but high in 'Simple Algebra'. Nevertheless, he has the same mark in Algebra as the pupil who is in the opposite situation concerning these variables. Then, compensation between variables is possible. Pupils with a lower level in 'Understanding' can practise more and they can get equal results in this way. If there are compensations of this kind, it is to be expected that the variable 'Understanding' has a slight beta-coefficient in our regression analysis, though it can be important when planning the learning process. A more precise analysis of this regression system would presuppose the investigation of learning processes.

We have discussed the task analysis in Chapter 2, and it was supposed that differences in aims may be necessary to get optimum learning results of all pupils. This cannot be investigated here and we can give only a description of the common tasks to get common cognitive aims. The most important aim is to have good knowledge of the previous topics. It is advantageous to have a positive attitude to the subject, but there is not needed any mathematical ability different from general scholastic ability.

Comparison of Weak and Bright Pupils

If there are differences concerning the optimal aims among the pupils these would be seen when comparing weak and bright pupils. A precise comparison presupposes a wider experimental design for this purpose. We are concerned here only to make a preliminary investigation which is connected with the task analysis presented before.

There were 17 pupils in this experimental group in the beginning of the investigation who did not pass Grades 6, 7, and 8 in the normal course. One of them was not passed because of prolonged illness. These 17 pupils did not take part in the final phase of the experiment and we have information about them only in Grades 6 and 7.

A pupil does not pass the Grade if he has got (with some exceptions) the mark 4 in two or three subjects. Most of the 17 failed students had the mark 4 in Swedish or German/English and only 5 of them in Algebra. There is no objective analysis of the reasons for not passing and we cannot analyse separately those five pupils who have not been passed because of Algebra.

In the remainder of the group there are also some pupils who have had a fail mark in Algebra, but they have been passed after taking a further examination during the summer holidays. Thus, there is no clear difference between the passed and the failed pupils concerning school success in Algebra. We do not restrict

ourselves so as to investigate only the failed pupils but we take a slightly wider group which we call "weak pupils". The criterium for a weak pupil is that he has got a fail mark (4) or the lowest pass mark (5) in the school report in Spring. When we investigate pupils in Grade 7, there are also some who have not passed. When we investigate pupils in Grade 9, this group has been selected, because these 17 pupils were no longer in the experimental group. We suppose that the 7 pupils who changed their schools during the experimental period did not belong to the number of weak pupils.

We shall compare these weak pupils with the set of bright pupils who have got the highest mark in Algebra (9 or 10). The frequencies of these extreme groups are in Table 5.12.

Table 5.12. The Numbers of Weak and Bright Pupils in Algebra

	Weak pupils	Bright pupils	Experimental group
Grade 7	20	29	140
Grade 9	19	24	119

We have determined the means of these extreme groups for the intervening variables presented in Table 5.11. For this abridged investigation we have combined the attitude variables to the variable 'Attitude to Algebra' presented before (p. 48). Besides, we have compared these groups as to their common school success by determining the corresponding values of variable 'Common School Success' (p. 49). Then, these scores have been transformed into standard scores. So we compare these groups with the total experimental group. These profiles for weak and bright pupils are presented in Table 5.13.

There are almost identical profiles in Grade 7 and in Grade 9. These profiles give in the main the same information as the regression analysis.

Table 5.13. Profiles for Weak and Bright Pupils in Algebra

Variable	Grade 7		Grade 9	
	Weak pupils	Bright pupils	Weak pupils	Bright pupils
Reasoning Ability	−0.6	+0.8	−0.3	+0.8
Numerical Ability	−0.5	+0.4	−0.4	+0.4
Attitude to Algebra	−1.3	+0.9	−0.9	+1.0
Simple Algebra	−1.2	+0.9	−0.9	+1.0
Understanding	–	–	−0.6	+0.6
Common School Success	−1.9	+1.0	−1.2	+0.9

This can be expected because of the linearity of regression. There is not, however, a complete symmetry with regard to the mean value, and the anomalies seem to have some significance. In the group of weak pupils, common school success is very low, especially in Grade 7. Thus, failing in Algebra is more closely related to weak performances in school in general than good results in Algebra with good performances. The situation is the reverse in the values for 'Reasoning Ability'. The weak pupils are closer to the mean value, especially in Grade 9, but bright pupils are very high on 'Reasoning Ability'. Thus, good results also presuppose high reasoning ability, but poor reasoning ability does not cause failure in Algebra.

This task analysis has not helped to solve the problem of what the reasons for failing are. We can only argue that it is no special problem of mathematics but a common problem of school teaching. The present writer has investigated all available material concerning these failed pupils and this confirms the previous result. According to the interviews and official documents it seems obvious that the failed pupils had weaker home environments than the average pupils in this school and they had difficulties in adapting to school work. Thus, the reasons for failing are outside the scope of our study.

Summary and Discussion

It has been our purpose to choose essential intervening variables from our test material for this experimental design. These were grouped beforehand as attitudes, abilities, and achievements (c.f. Figure 2.1). After the preliminary analyses presented in Chapters 3 and 4 we have in this chapter collated the results.

The forming of combined variables concern-

ing abilities was clear. There was in many analyses a reasoning factor and a numerical factor and their constancy coefficients were good. Corresponding combined variables were formed by adding the weighted scores of the tests which were highly loaded in these factors. The combined variable 'Reasoning Ability' connects reasoning and visual tests. This variable includes inductive and deductive reasoning and its nature cannot be simply described. The nature of the variable 'Numerical Ability' has already been analysed by Werdelin before.

We have used in Grade 9 one more combined variable 'Understanding' which we list as an ability variable. It was determined by tests which were presented only in Grade 9, and it has a correlation coefficient of .59 with 'Reasoning Ability'. These tests are not common among the ability tests and the present writer made his own interpretation for this variable: This variable is connected with performance at the higher levels when this performance is newly learned. A more precise analysis of this variable is still needed. It may be that 'Understanding' is a situational variable (c.f. Wallen and Travers, 1963, p. 491) which has no common meaning when describing crystallized abilities.

The role of these ability variables is not very important when using as criteria the beta-coefficients. Instead of that, 'Reasoning Ability' and 'Understanding' correlate moderately with the dependent variables. It may be that they have affected the other variables ('Attitude to Algebra' and 'Simple Algebra') and in this way success in Algebra.

It was possible to form one combined variable concerning the affective attitude to Algebra or to use three separate variables in this domain. The cognitive component of attitudes has been combined with common school success in this investigation and we have omitted it. The variable 'Common School Success' was formed using the average marks but we have not used it here, either as an intervening variable or as a dependent variable.

In the total analysis of variables the attitude variables were not loaded in the same factors as the ability variables, but there are perhaps developmental connections between these groups of variables. The affective attitude to Algebra remains almost unchanged during the experimental period. Thus, we have used the combined variable 'Attitude to Algebra' as an intervening variable.

The combining of attitude variables is unsure because they have been formed using different techniques and their internal structure has not been analysed perfectly. All the variables 'Pleasantness of Algebra', 'Attitude to Success in Algebra' and 'Affective Attitude to Algebra' correlate rather highly with the 'Mark in Algebra' (> .50) but only the variable 'Attitude to Success in Algebra' has a significant beta-coefficient in Grade 7. Thus, only this variable includes information which is essential to the dependent variable and which is not included in other intervening variables. In Grade 9 the variable 'Pleasantness of Algebra' has almost the same coefficients as 'Attitude to Success in Algebra'. The variable 'Affective Attitude to Algebra' has smaller beta-coefficients than the others. According to these results we may conclude: The real affective attitude to Algebra is not very important when predicting success in Algebra. Instead of that, the pupils' feelings concerning their success in Algebra are important when making predictions. There is a high constancy in marks, and so, these feelings remain almost unchanged.

The achievements in algebra-tests are connected both with reasoning factors and attitude factors in the total analysis. Besides, these are connected with common school success. In Grade 9 there is already a special factor for simple achievements in Algebra (factor V in Table 5.6). We have formed an intervening variable by combining the test variables 'Algebra I', 'II', and 'V'. This variable is called 'Simple Algebra'. This variable has the highest correlation coefficients with the dependent variables and it is the best predicting variable both in Grade 7 and in Grade 9. This is natural in Grade 7 when the success in Algebra is connected closely with this kind of simple task. Perhaps we may not yet take this variable as an intervening variable in Grade 7. In Grade 9 the tasks are more complex but the predicting value of 'Simple Algebra' remains unchanged. It may be argued that there is a delay in tasks as in the results of Dahllöf (1967). The present writer has drawn the following conclusions: The variable 'Simple Algebra' has been formed as a

result of pupils' abilities, attitudes, personality traits etc. It includes much of the variance which is common in all school learning. It is the only intervening variable of this character. That is why its meaning is here more important than it should be when thinking only of the meaning of simple algebraic tasks.

We have used 'Complex Algebra' as a dependent variable because it is a pure achievement variable. Its regression analysis is of a similar nature as when using 'Mark in Algebra' as a dependent variable. In both cases we can sum up the results: There is no need of mathematical ability different from general scholastic ability. The role of a separate affective attitude is also slight but general feelings about success in Algebra predict success in Algebra better. Both abilities and attitudes affect the previous results in Algebra. In this way we can predict the future results well.

We have also compared the results of weak and bright pupils. It gave us only slight new information. It may be that reasoning ability has more effect among bright pupils than among weak pupils. The effects of attitudes seems to be the opposite. These results are reliable only if the scales of these variables are perfect interval scales. This anomaly will be caused also if the variable differentiates well at one end but not at other end.

We have tested the linearity of the regression. Nevertheless, it is not evident that we can take a linear regression model. It may be that the real connection between some variables is nonlinear. It may be that understanding has no effect among weak pupils but it is more and more important when the results become better. All these nonlinear connections could be better investigated only after more precise analysis.

Chapter 6. DIFFERENCES BETWEEN THE TWO TEACHING GROUPS

Comparison of Groups

Two teaching groups were formed at the beginning of the experimental period, as is reported in Chapter 2. These groups should not differ essentially from each other according to the preliminary estimate. We can check the comparableness of these groups afterwards when we present the means and standard deviations (S.D.) of the variables in Table 5.7 for both groups separately. These are in Table 6.1 for the variables which were obtained in Grade 7 (Grade 6).

The only significant difference is between the means in 'Reasoning Ability'. It is to be seen in the distributions of scores that in Group 2 there are more pupils who are weak in reasoning. The difference in 'Common School Success' is almost significant. Thus, there may be slight differences between these groups in some variables, but no essential differences concerning mathematics specifically. We must try to evaluate the meaning of these differences when comparing the results of these groups.

Table 6.1. Comparison between Groups in Grade 7

Variable	Group 1 (N = 60)		Group 2 (N = 59)		Significance of difference between means	
	Mean	S.D.	Mean	S.D.	t	
Reasoning Ability	60.1	13.7	52.6	16.7	2.7	$p < 0.01$
Numerical Ability	54.5	9.8	52.2	12.4	1.1	$p > 0.05$
Attitude to Algebra	63.6	11.8	66.0	11.5	1.1	$p > 0.05$
Common School Success	80.3	7.3	77.3	7.6	2.2	$p < 0.05$
Simple Algebra	45.0	8.1	42.3	10.5	1.6	$p > 0.05$
Mark in Algebra	7.6	1.4	7.2	1.4	1.6	$p > 0.05$

Differences in the Algebra Syllabus

There were slight differences between these groups in the Algebra syllabus and these were checked by using the pupils' written exercises. A description of the courses appears in Appendix A. We will describe here only the mean outlines of the differences.

The total number of exercises was greater in Group 1 than in Group 2. This does not mean differences in the amount of training, for the exercises in Group 1 were simpler than those in Group 2. The process of teaching was almost similar. Because of simple exercises at the beginning of a new topic, the teaching was more inductive and less teacher-centred in Group 1 than in Group 2, where the proof of the rules came first and then these were practised.

The essential differences in the number of written exercises are presented in the following list:

	More in Group 1	More in Group 2
Grade 7	Addition, subtraction and multiplication of polynomials (Algebra VI, VII, VIII)	Exponential laws and exercises (Algebra IX) Equations (Algebra XI)

	More in Group 1	More in Group 2
Grade 8	Applications of equations (Problems II, III) The coordinate system and functions (Algebra XIV)	Operations with polynomials (Algebra VI, VII)
Grade 9	Polynomials and exponential expressions Functions (Algebra XIV)	Verbal problems (Problems II, III)

There are also differences in the number of exercises concerning topics which have not been tested in this investigation. These are connected in the main with the modernised algebra (sets, functions, elements of logic in connection with equations) which was included in the Group 1 course.

Differences in Factor Structure

The factor analyses in Chapters 3, 4, and 5 were made for the total experimental group. The differences in the factor structure caused by the independent variables (differences in the teaching) were omitted then. It would be possible to make separate factor analyses for these two groups and then compare the factor structures, but it would be bothersome and we will now afterwards check if there are essential differences in the factor structure of these groups in two situations.

The first situation is connected with the total analysis presented in Chapter 5. When comparing the factor structures of these groups we have used subgroup analysis presented by Heinonen (1968). First an ordinary factor analysis is made for the total experimental group. For every pupil factor scores are then calculated for these factors by means of Lederman's method. Then, the experimental group is divided into subgroups. The correlation coefficients between test variables and factor scores is determined separately in each subgroup. We have all the time the same factor configuration as the background and we can now compare the corresponding coefficients in these subgroups. If there are differences in coefficients, this represents differences in factor structure. Heinonen (1968) has presented several methods for this comparison.

If the subgroups have been selected in the same way from the same population, this method gives possibilities for evaluating the reliability of factor structure. Then, the differences in coefficients are caused by stocastic variance. If we know that the subgroups are not selected from the same population, this method gives us possibilities for evaluating the differences in factor structure which are caused by the differences in subgroups. Then, we must suppose that the reliability of factors is high enough to make this comparison possible.

We have used this method when comparing the factor structure between Groups 1 and 2 in the total analysis in Grade 9. We have used here a rotation with four factors because these include already enough information for this analysis. The rotated factor matrix is presented in Appendix B (Table 6.2). The factors can be named as factors I–IV in Table 5.6 though they are not exactly identical:

Factor I. *Reasoning factor.*

Factor II. *Observed behaviour indicating interest in school work.*

Factor III. *Affective attitude to Algebra and specific school success in Algebra.*

Factor IV. *Numerical factor.*

Here the tests which were loaded in the *Factor of understanding mathematical principles* (Factor VI) in Table 5.6 are now loaded mostly in the *Reasoning factor.*

We have determined the factor scores of each

pupil for these factors in Table 6.2 by means of Lederman's method. This method does not give exactly correct values. Thus, the correlation coefficients between the factors still have to be determined.

The second task is to determine the correlation coefficients between the test scores and factor scores in Group 1, in Group 2, and in the total experimental group. These are presented in Table 6.3 (Appendix B). The matrix for the total group should be identical with the matrix in Table 6.2 and the differences are due to the approximation method used when we estimate factor scores. The correlation matrices between the factors are found in Table 6.4 (Appendix B). All these correlation coefficients are low and the coefficients in Table 6.3 correspond well with the original factor loadings.

The comparison of subgroups is made in Table 6.5 (Appendix B). There are differences (d) between the coefficients in the two subgroups (Group 1 minus Group 2). The sum of the squares of these differences is then (Σd^2). The differences can be interpreted as difference vectors on the factor line. Because these factors, and also the difference vectors, are almost orthogonal, the sum Σd^2 indicates the square of the total difference vector for each test variable.

The greatest differences between the coefficients in these subgroups are in tests No. 13, 'Cubes' and No. 10, 'Multiplication'. Most of the differences are close to zero and they do not cause changes in the interpretation of factors in the subgroups. There are no clear differences in the coefficients for tests of algebraic performances, which would indicate that there are systematic changes in factor structure as a result of differences in teaching.

The differences in test variables 'Cubes' and 'Multiplication' cannot be consequences of the differences in teaching. We have no test to indicate the significance of differences and it may be that all these coefficients are not yet significantly different.

As a summary, there are no essential differences in the factor structure of algebraic performances between these two groups. There are no special differences in the loadings of factor IV *(Numerical Factor)* and then no differences in automatization of performances. The factors for all groups in Table 6.3 can be interpreted like the factors in Table 6.2. According to this result, Groups 1 and 2 can be samples taken from the same population. The factor structure is very stable between different groups, as was verified earlier between different Grades.

The second situation for the comparison of factor structures is taken in Grade 8. During the training programme there were many situations where the pupils in Group 1 had more exercises than those in Group 2 or *vice versa*. The comparison of the structures between the groups will then give information about the effects of the training programmes in this phase. An interesting difference in training appeared at the beginning of Grade 8. Already in Grade 7 the pupils in Group 1 had studied the simplification of such algebraic expressions as in the test 'Algebra VI', and they had this test for the first time after the teaching period. This test was repeated after the summer holidays (3 months) without any practice meanwhile. The pupils in Group 2 began to study this topic at the beginning of Grade 8. After that period this group was also tested.

The number of written exercises before the test was almost equal for both groups. Group 1 had less concentrated training and had lapses of memory. Thus, the level of performance was lower in Group 1 than in Group 2 in Table 6.7. There were many differences between these groups as to the form of the training practice and it may be that this has caused differences in the structure of the tasks.

We have investigated this hypothesis using the regression analysis as in Chapter 5. The test 'Algebra VI' is the dependent variable and the intervening variables presented in Table 5.7 are independent variables. We can use here the variable 'Common School Success' as an independent variable because the results in Algebra have not caused differences in 'Average Mark' during this short training period. Besides, it is interesting to know the role of common observed behaviour when predicting specific knowledge of Algebra. Regression analysis is made separately in Group 1, in Group 2 and in the total experimental group. The results are in Table 6.6.

Most independent variables have a significantly positive correlation with 'Algebra VI'

Table 6.6. Regression Analysis for 'Algebra VI', Grade 8

Independent Variables	Group 1 (N = 60) $r_{y \cdot n}$	Beta	Significance for Beta	Group 2 (N = 59) $r_{y \cdot n}$	Beta	Significance for Beta	Total (N = 119) $r_{y \cdot n}$	Beta	Significance for Beta
Reasoning Ability	0.35	0.11	$p > 0.05$	0.41	0.19	$p > 0.05$	0.22	0.00	$p > 0.05$
Numerical Ability	0.06	−0.15	> 0.05	0.24	0.08	> 0.05	0.07	−0.09	> 0.05
Common School Success	0.51	0.33	< 0.05	0.38	0.10	> 0.05	0.32	0.14	> 0.05
Attitude to Algebra	0.44	0.28	< 0.05	0.45	0.20	> 0.05	0.44	0.34	< 0.001
Simple Algebra	0.47	0.11	> 0.05	0.48	0.16	> 0.05	0.34	0.11	> 0.05
Multiple Correlation	0.61	$p < 0.05$		0.55	$p < 0.05$		0.48	$p < 0.05$	

Significance test: the t test

but the prediction is made faulty because multiple correlation coefficients are not high. Correlation coefficients and beta-coefficients are almost equal in both groups. The greatest difference is in the variable 'Common School Success'. This variable is more important in Group 1 than in Group 2 in prediction. There is a reverse difference between these groups concerning the variable 'Numerical Ability' though the coefficients did not differ essentially from zero in each group. Pupils in Group 1 were tested after the summer holidays and then perhaps common factors like memory, ability of concentration, etc., have more effect there than in Group 2, where the test was given after an exercise period. The greater correlation coefficient of 'Numerical Ability' in Group 2 may indicate that there is already automatization of performance. The low values of multiple correlation coefficients indicate that the test 'Algebra VI' has much specific variance. That is why we cannot conclude whether there are different structures of performance in Group 1 and Group 2.

Differences in Achievement Levels

We can compare results in these groups by using the means of scores in achievement tests. The comparison is easier if we present the means as percentages of the maximum score. Those results for all algebra-tests are in Table 6.7. The month of testing is also given and here we can make a more precise comparison between the different presentations of a test.

Table 6.7. Means of Scores in Algebra-tests in Groups 1 and 2 as Percentages of the Maximum Scores

Test	Group 1 Grade 7	Group 1 Grade 8	Group 1 Grade 9	Group 2 Grade 7	Group 2 Grade 8	Group 2 Grade 9
Algebra I	74 Sep	70 Sep	80 Jan	72 Oct	71 Sep	74 Jan
Algebra II	82 Oct	72 Sep	83 Jan	80 Oct	68 Sep	80 Jan
Algebra III	36 Oct	31 Sep	48 Jan	31 Oct	35 Sep	45 Jan
Algebra IV	43 Oct	—	56 Jan	40 Oct	—	48 Jan
Algebra V	56 Nov	—	65 Jan	49 Nov	—	60 Jan
Algebra VI	64 Dec	59 Sep	86 Jan	—	82 Oct	72 Jan
Algebra VII	33 Jan	33 Sep	60 Jan	—	—	42 Jan
Algebra VIII	39 Apr	—	53 Jan	37 Nov	—	47 Feb
Algebra IX	34 May	26 Sep	50 Jan	44 Jan	39 Sep	49 Mar
Algebra X	30 May	24 Sep	59 Jan	—	—	48 Feb
Algebra XI	—	64 Oct	72 Feb	43 Mar	{39 Sep {75 May	72 Feb
Algebra XII	—	—	33 Jan	—	—	36 Feb
Algebra XIII	—	—	68 Mar	—	—	63 Apr
Algebra XIV	—	—	75 May	—	—	65 May

Table 6.8. Means of Scores in the Tests 'Equations' and 'Problems' as Percentages of the Maximum Scores

Test	Group 1			Group 2		
	Grade 6	Grade 7	Grade 9	Grade 6	Grade 7	Grade 9
Equations	45 Feb	39 Sep	46 Mar	36 Feb	32 Sep	45 Mar
Problems I	48 Feb	47 Sep	62 Jan	46 Feb	52 Sep	59 Jan
Problems II	67 Feb	68 Sep	74 Jan	60 Feb	64 Sep	72 Jan
Problems III	—	—	56 Apr	—	—	59 Apr

The corresponding results in the tests 'Equations' and 'Problems' are given in Table 6.8. We presented some of these tests twice in Grade 6 and the results of the second test are given here.

In Grade 7 and Grade 8 there are differences in achievements which are directly connected with the differences in the training programme. In Grade 7 there were better results in Group 1 in tests 'Algebra VI' and 'Algebra VII', and correspondingly in Group 2 in tests 'Algebra IX' and 'Algebra XI'. In Grade 8 the performances in the test 'Algebra VI' were essentially better in Group 2 because of the long training period of polynomials.

We have compared the achievements in Grade 9 more precisely in Table 6.9. Here are the means and standard deviations (S.D.) of each test variable separately for both groups. Then, the significance of the difference between means has been tested using the t-test. The test variables have been listed according to their t-values. With the exception of the tests 'Algebra XII' and 'Problems III' the means are higher in Group 1 than in Group 2 but the difference between the means is highly significant only in tests 'Algebra VI', 'Algebra VII', 'Algebra X', and 'Algebra IV'. These tests include in the main items which were trained in Group 1 during Grades 7 to 9, but in Group 2 only in Grade 8. The total amount of training cannot be compared well because many difficult exercises include training connected with those tasks. If we take only those exercises listed in Appendix A for these tests, then Group 1 has more training (in addition about 450 exercises) than Group 2 (in addition about 340 exercises).

The tests 'Algebra IV', 'VI', 'VII', and 'X' include items which are of medium difficulty in

Table 6.9. Comparison between Achievements of Group 1 and Group 2 in Grade 9

Test	Group 1 (N = 60)		Group 2 (N = 59)		Significance of Difference between Groups	
	Mean	S.D.	Mean	S.D.	t	
Algebra VI	17.2	2.9	14.5	3.0	5.0	$p < 0.001$
Algebra VII	7.1	2.6	5.0	2.2	4.8	< 0.001
Algebra X	10.6	3.5	8.5	3.1	3.4	< 0.001
Algebra IV	6.8	1.5	5.8	1.8	3.3	< 0.001
Algebra XIV	8.9	2.1	7.7	2.2	3.0	< 0.01
Algebra I	16.0	3.0	14.8	3.5	2.0	< 0.05
Algebra VIII	9.5	3.4	8.5	3.4	1.6	> 0.05
Algebra V	15.7	5.0	14.3	5.1	1.5	> 0.05
Problems I	7.4	1.1	7.1	1.2	1.4	> 0.05
Algebra XIII	4.7	1.1	4.4	1.4	1.3	> 0.05
Algebra III	8.7	3.5	8.0	3.4	1.1	> 0.05
Problems II	7.4	1.1	7.2	1.3	1.0	> 0.05
Algebra II	16.6	3.2	16.1	2.9	0.9	> 0.05
Algebra IX	9.9	3.1	9.7	3.1	0.4	> 0.05
Equations	11.0	6.5	10.8	6.3	0.2	> 0.05
Algebra XI	10.7	2.5	10.7	2.5	0.0	> 0.05
Algebra XII	2.3	1.8	2.5	1.7	0.6	> 0.05
Problems III	6.7	2.2	7.1	2.6	1.0	> 0.05

these courses. There are from 2 to 5 operations in each item and all of these operations were trained separately before these tasks were presented. There are no highly significant differences in simple algebraic tasks as in tests 'Algebra I' and 'Algebra II'. This may be caused by the ceiling effect which can be seen in the distributions of scores of these tests in Grade 9, but this may be also caused by the fact that these tasks were trained to an equal extent already in Grade 7 in both groups. Nor are there any essential differences between the numbers of exercises connected with these tests.

The most difficult tasks concerning the simplification of expressions were in the test 'Algebra XII' and partly in the test 'Algebra IX'. These were taken directly from the training programme of Group 2 and they were only slightly trained in Group 1. The number of exercises concerning the test 'Algebra XII' did not differ essentially but there was a difference in the difficulty of the exercises. There were, however, no differences in results. The training of exponential expressions was divided into many parts in Group 1 while it was concentrated into a special period in Group 2. It is difficult to compare the number of exponential exercises because these are included also in other tests. The special training was greater in Group 2, especially concerning the more difficult expressions. However, the results did not differ essentially.

Group 1 had training in simple equations already in Grade 6. This was compensated in Group 2 by more training in Grades 7 and 8. At the end of the experimental period there were no essential differences in the results of the tests 'Equations' and 'Algebra XI'. The training and the results in complex equations ('Algebra XIII') were almost equal in both groups.

The training of problems began already before Grade 6 and it is difficult to evaluate the training concerning the tests 'Problems I' and 'Problems II'. There were more exercises for the test 'Problems III' in Group 2, but no essential differences in results were discovered.

The importance of forgetting can be seen when we compare the results of tests that have been repeated without training in the interval. The greatest drops in performance level were in the test 'Algebra II' from Grade 7 to Grade 8, if we compare the mean values in percentages. Later this level again increased because of training more complex tasks. There was also a great drop in the test 'Algebra VI' in Group 2 from Grade 8 to Grade 9. There was intensive training in Grade 8 but no special repetition of those exercises in Grade 9. During the same time interval there was a repetition in Group 1 which caused a high increase in results.

In Grade 6 there was an experimental period when the pupils in Group 1 learned to solve problems, like those in the test 'Problems II', by using equations. The pupils in Group 2 solved the same tasks by means of unit values, a method which was known to them already (Malinen, 1961). The pupils in Group 1 used their new method in February, Grade 6, when this test was presented. In the retests (September, Grade 7, and January, Grade 9) they again used their earlier method. The change in the method did not, however, make them less able. This training in the application of equations in Grade 6 did not even help them to solve more complex applications of equations, such as 'Problems III' in Grade 9.

There is a ceiling effect in the results of those tests where the mean score is high. For these tests the mean does not give appropriate information about the results. We use yet another comparison, which also suits these tests. We are particularly interested in those pupils who have failed in the tests and the comparison of those pupils is as important as the comparison of means.

In order to see the number of failures we must take some boundary for accepted performance. The boundaries for the minimum aims have not been much discussed. There are opinions that the number of correct answers should be greater than 90 per cent for simple tasks in arithmetic (Magne, 1967, p. 57). We have taken a boundary of 50 per cent for simple algebraic tasks because the tests were time limited so that many pupils did not reach the end in the tests 'Algebra I', 'Algebra II' and 'Equations'. For complex tasks we have taken a boundary of 33 per cent or 25 per cent. With such low pass scores we cannot be sure that the pass pupil has understood all ideas, but on the contrary we can be sure that the failed pupil really has defects in the learning of this topic.

The frequencies found for the failed pupils in Grade 9 are shown in Table 6.10. In all cases, except the test 'Algebra XII', the frequencies are greater in Group 2. The difference between the percentages of failed pupils is, however, significant ($p < 0.01$) only in the test 'Algebra VII' and almost significant ($p < 0.05$) in the test 'Algebra VI'. Failing in simple tasks is closely connected with failing in complex tasks, which can be expected because of the high linear correlation between these variables. Thus, the number of failed pupils does not provide new information as to the comparison between groups.

Table 6.10. The Frequencies of Failed Pupils in Grade 9

Test	Frequency Group 1	Frequency Group 2	Boundary for Acceptance in Percents
Algebra I	2	8	50
Algebra II	2	3	50
Algebra III	16	23	33
Algebra IV	15	21	33
Algebra V	4	7	33
Algebra VI	5	13	50
Algebra VII	11	26	33
Algebra VIII	12	17	33
Algebra IX	8	11	33
Algebra X	15	17	33
Algebra XI	9	6	50
Algebra XII	25	20	25
Algebra XIII	6	7	33
Algebra XIV	6	10	33
Equations	8	13	33
Problems I	1	3	33
Problems II	2	4	33
Problems III	6	6	33

In this situation we have taken the boundaries presented before as a criterion for acceptance. The aims have been badly fulfilled especially concerning the topics connected with the tests 'Algebra III', 'Algebra VIII', 'Algebra X', and 'Algebra XII'. To reach the aims better we have two ways: (1) We can make the aims lower. (2) We can teach these topics more effectively. It is the present writer's opinion that we cannot make the aims lower in simple tasks like tests 'Algebra III', 'Algebra IV', and 'Algebra VI'. For these tasks more effective teaching is needed. The complex tasks may not be necessary for all pupils. It seems to be a waste of time to take exercises as in the test 'Algebra XII' when more than 35 per cent of pupils complete only one or none of these items after this training.

Summary and Discussion

These comparisons between groups have practical consequences when planning the Algebra curriculum. However, we have not drawn conclusions here as to whether the one course of Algebra is better than the other. An investigation of the curriculum presupposes more detailed comparisons than we can make. When we have studied the effects of independent variables (differences in external inputs), we have not dealt with individual differences, i.e. intervening variables. Only when we have studied these two groups of variables together can we make more detailed conclusions about the effects of different material during instruction.

The differences between the two courses during the experimental period were slight and there were no clear differences in the structure of performance between these two groups. In the comparison of structures we have used two methods: subgroup analysis presented by Heinonen, and regression analysis. These methods have different aims. Whether the factors in the subgroups have the same content as in the total group will be investigated in subgroup analysis. There were no essential differences between the structures of Group 1 and Group 2 in Grade 9 in this investigation. The regression analysis was made in Grade 8 to determine if the structure of the test 'Algebra VI' is different in these groups because of the differences in training. This analysis was made in the system of our intervening variables. Only slight changes in coefficients were to be seen.

The comparison of performance levels using our achievement tests presupposes that these variables are measured by means of interval scales before and after training. This requirement has not been fulfilled completely. Thus,

the comparison of achievements is partly defective. That is because we have used yet another idea: pupils who know some task well can all reach the maximum score. Then, absolute aim can be discussed. Both these methods of investigation are necessary in didactics, but we cannot use them both in connection with the same tests. In our situation the comparison of means is more important.

In many algebra-tests there are better performances (higher means and fewer failed pupils) in Group 1 than in Group 2. This is natural, because this group had a larger number of simple exercises. But in addition, Group 1 is not left behind Group 2 in complex tests, either, though these were more common in the Group 2 syllabus. The skill was transferred from simple exercises to complex exercises but not *vice versa*. Besides, pupils in Group 1 had more topics outside the test programme as presented in Appendix A. We have varied the external inputs of these groups by using different kinds of exercises. This has an effect on dependent variables: pupils in Group 1 have, on the average, learned more mathematics. The advantage of simple exercises cannot be precisely determined because there are also other independent variables affecting the results.

A longer training time has given better results. This can be seen in the results of the tests 'Algebra VI' and 'Algebra VII'. Pupils in Group 1 practised them during three years while the pupils in Group 2 had this training mainly in Grade 8. Group 2 was better in Grade 8, but after that it remained behind the other group. Another example is the simplifying of exponential expressions. In Grade 7 the pupils in Group 2 practised many exponential expressions as in the test 'Algebra IX', but they lost their lead during the experimental period though, altogether, pupils in Group 1 had not so much special training in exponents. We have here another difference in the external information input, but this difference also is connected with other variables in the total experimental design. We do not have pure situations here where the same number of exercises has been presented during a short period and during a long period. However, it seems more advantageous to have many short periods with repetition than one systematic presentation of the total topic.

We have tested the comparability of these groups at the beginning of this chapter. There were significant differences in 'Reasoning Ability' and 'Common School Success'. There are more weak pupils in Group 2 and this may partly have caused the differences between the results of these groups. The role of 'Reasoning Ability' was not high when predicting the 'Mark in Algebra' but it is closely connected with the general capacity for work in school. When investigating the differences in the external information input, as presented before, the reverse situations would also be needed: e.g. simple exercises in Group 2 and complex exercises in Group 1. This is difficult to realize in practice because the complete separation of different topics is not possible. The experimental design must be planned according to two different Algebra syllabuses and all independent variables (differences between courses) will work together.

The didactical results presented before are not unknown to experienced teachers, nor to the writer of the text book, because he advises teachers to use repetition and more simple exercises (Väisälä, 1960, pp. v–vi). The realisation of the instruction is, however, very dependent on the text book, and it is our conclusion that the teaching must be planned more in accordance with aspects of effective learning than was done in Group 2. The base which has been obtained from the systematical presentation of algebraic facts is not sufficient for the planning of teaching in these Grades.

Chapter 7. MAIN RESULTS AND DISCUSSION

Structure of Information Output Variables

Many analyses concerning the structure of our information output variables have already been made. On the basis of these studies, our variables were first divided into cognitive domain and affective domain variables. In the cognitive domain, most tests could be classified beforehand as reasoning, deductive, visual, numerical, or achievement tests. All the ability and achievement tests were analysed together. Besides, there were some tests which were not connected with specific topics in the curriculum, nor with the known ability structure. The structure in the affective domain was known beforehand to be faulty. Thus, structure analysis was needed in our situation.

We have analysed the structure of our test variables by factor analysis. There were difficulties in making decisions about the number of factors. We have used tw^ methods: (1) We have taken factors so that almost all essential information is included in the rotation. (2) We have taken factors so that only the most important information is included in the rotation. When using the first method we have obtained some factors which have been defined by few variables. When using the second method we have obtained a very rough description of the structure. In spite of that, the latter way has seemed more useful for the subsequent treatment of the material. When using the same tests in Grade 7 and Grade 9 we have taken an equal number of factors.

In most factor analyses we have used the Varimax rotation, and only in Grade 9, for the larger test battery, also the oblique analytic cosine rotation developed by Ahmavaara and Markkanen. It is the present writer's opinion that the latter method gives more precise location of factors, and so it would make the interpretation easier. Thus, analytic cosine rotation can be used as a control for other methods. In our situation, we have not found essential differences between the factors of these methods of rotation.

The factor structure remains almost unchanged in transformation from Grade 7 to Grade 9. These transformations were made separately for the cognitive domain variables and for the affective domain variables. There have also been other ways of investigating variations in structure. In Chapter 3 development analysis was used. Then, many test variables in the cognitive domain in Grades 7, 8, and 9 were analysed in a common space. This offered better possibilities than transformation analysis for giving information about the changes of test variables. The variations in factor structure were further studied in the subgroup analysis in Chapter 6. This method is suitable when comparing the factor structures in two different subgroups.

We have here spent much time in studying the changes in variables and factors, because there are few studies where the same pupils have been tested after two years with the same extensive test battery. It would have been more

Cognitive domain factors:	Affective domain factors:
Numerical factor	*Affective attitude to Algebra*
Visual-reasoning factor	*Interest in Algebra and other school work*
Factor of school success in Algebra	*Dissatisfaction with results in Algebra*
Factor of understanding mathematical principles	*Aspiration level of Algebra*
	Attitude to Algebra syllabus
Factor of complex Algebra	*Common school success*
School reasoning factor	
Algebra and induction (visual) factor	

effective for this study if already in Grade 7 there had been more tests. Now all factors could not be determined well in Grade 7.

Our analyses gave the following factors (with the exception of the factors given in the development analysis and in the total analysis). The first factors are precisely defined and the last factors imperfectly defined.

The same name for factors has been used in separate analyses though the content of the factor has not been precisely identical. There was no detailed analysis of all these factors. A common analysis of the most important factors was made in Chapter 5.

There were only two typical ability factors in the cognitive domain. In particular, there was no special visual factor. In the test battery three of Werdelin's visual tests were included, but in our experiment they were connected with the reasoning (deductive) tests. Many pupils had difficulties in understanding the idea presented in the test 'Cubes'. Thus, for these pupils it has been a reasoning test as to its nature. The other visual tests have also mixed with the reasoning tests in these batteries where there were many achievement tests, too.

In this phase the cognitive and affective domain variables are still separated. This was done mostly for practical reasons, because the number of variables was great, but also the interpretation of factors has been clearer in this way. In the second phase (Chapter 5) we have made the important variables a subject for further analysis. This total analysis of pupils' capabilities gave four well-defined factors: *Reasoning factor*, *Numerical factor*, *Affective attitude to Algebra and specific school success in Algebra*, and *Observed behaviour indicating interest in school work*. This analysis also included variables which were taken later as dependent variables. The interpretation is not made by means of the intervening variables alone, which have been indicators of pupils' capabilities. The most interesting result here was that the ability and attitude variables were not loaded in the same factors, but both were connected with the achievement variables.

In the subsequent phase we have formed the intervening variables. This has been made on the basis of the present writer's intuition and it can therefore be criticised. It was the intention to form "pure" variables, but our factors did not fulfil this condition. Our variables have been measured by means of different techniques, but this does not prevent them from combining in this phase. Instead of that, the internal structure of many variables was not clear and we have made a content analysis of the variables and aims in teaching Algebra. This analysis led to the following structure of information output variables:

Dependent variables:
- Mark in Algebra (Grade 7 and Grade 9)
- Complex Algebra (Grade 9)
- Algebra VI (Grade 8)

Intervening variables:
- Reasoning Ability
- Numerical Ability
- Attitude to Algebra
- Simple Algebra
- Understanding (only in Grade 9)

The variable 'Attitude to Algebra' was divided into three separate variables in a more precise analysis. The variable 'Common School Success' was taken as an intervening variable when predicting the variable 'Algebra VI' in Grade 8.

The intervening variables are heterogeneous in structure. 'Simple Algebra' is a learning result where all other intervening variables have influenced. The greatest weakness in our structure is that the separation into these dependent and intervening variables is logically not quite correct. The growth of attitudes has occurred during many years in connection with the success in mathematics. The variables 'Mark in Algebra' and 'Attitude to Algebra' have the same background. Attitudes may be taken as intervening variables only when investigating new processes in mathematics.

Structure of Performance in Algebra

In our study it was not possible to analyse learning processes in Algebra. The teacher has given information to the pupils; they have worked with it, and afterwards we have measured the performance. Our differential method gives us opportunities for analysing in what way differences in performance can be predicted from differences in intervening variables. If this is a real prediction system, there is a causal relationship between intervening and dependent variables. It would be more advantageous to take as independent variables those variables which arise from the study of learning processes, but we cannot study well those situational variables. Our prediction has a more formal meaning without clear causal relationship, but this is a first look at the structure of performances in Algebra.

The multiple correlation coefficients are between .78 and .84 when predicting 'Complex Algebra' and 'Mark in Algebra' in this system. The inclusion of other independent variables like these would probably not have increased these coefficients essentially. The most important variables in our prediction were 'Simple Algebra' and attitude variables. The importance of 'Reasoning Ability' was slight and the importance of 'Numerical Ability' insignificant. Besides, weak attitudes were connected more closely with weak preformances in Algebra than high attitudes with high performances, especially in Grade 7. High reasoning ability was connected closely with high performances, but the pupils with a weak performance were not especially low in reasoning.

The meaning of the variable 'Understanding' has remained unclear. It is moderately important in Grade 9 when predicting performances both for weak and bright pupils. We have some clues that it is important in the first phases of learning, but later on, performance has been established almost independently of it.

The variable 'Simple Algebra' and the attitude variables are all closely connected with the previous situations in school. It would be misleading to say that mathematical ability is insignificant for the learning of Algebra in this experimental group. General reasoning has an effect here, but it works in connection with common skill and interest in school subjects. We do not need to speak about a special mathematical ability which would influence school success in Algebra, and common réasoning ability mainly helps indirectly. Instead of that, there is a variable for a specific attitude to Algebra. A complete study of attitudes is not possible here in the frames of a one-pupil system, because attitudes arise from social interactions between the pupil and his environment.

We cannot proceed any further in the frames of this study, but we can discuss future opportunities. The studies of learning have nowadays been more interested in individual differences between pupils. It should be possible to construct theories concerning the learning of able and less able pupils. In the same way it may be possible to take into consideration differences in understanding. These would be useful, but as such they do not help the teachers much, unless they are connected with the teaching situation. Situational variables seem to be much more important than pure ability variables when predicting performances of algebra. Thus, teaching theories are needed to describe specific teaching situations. In those theories, different aims must also be taken into consideration.

Our investigation was made in a secondary school and the pupils belonged to the upper half of the age group as to their skills in mathematics and we cannot estimate the results for the whole age group. To a great extent this group was selected according to reasoning ability and for this reason it has perhaps had too small a role. As a result of this the importance of attitudes may have increased when predicting success in Algebra.

Didactical Implications

In Chapter 1 we have presented some didactical problems which are topical for school reforms in Finland. Our experimental design was not especially planned to study these problems. The main interest has been concentrated on analysing differences between pupils which would be of interest when planning teaching. However, we have presented in previous chapters some results which are closely connected with the syllabus of Algebra.

1) It was advantageous, especially for weak pupils, to use simple exercises and the inductive method. The training of difficult problems, expressions and equations did not greatly help weak pupils. It was possible to reach elementary skills in equation solving and in simplifying expressions by means of simple exercises. Then, time can be saved for other topics of Algebra. It made the teaching more effective that we used the method of repeated teaching situations according to the spiral principle.

2) In predicting success in Algebra it was found that the most important mediating variables were 'Simple Algebra' and attitude variables. The differences in 'Understanding' seem to be moderately important. All these differences are also between weak and bright pupils, though there were differences in weights. The parallel courses in "level grouping" differ from each other according to the plans presented by Lyytikäinen (1967) in two respects: in their extensiveness and in their operational-theoretical dimension. The differences in understanding, reasoning ability, and common interest in school work suit well these differences between the courses. We can measure those differences well in order to give information to the parents, who decide the choice. The affective attitude may be best included in the selection process if the pupils and parents choose the "level group". We have presented many dimensions which are important in selection, but their interaction is unknown. Thus, it is impossible to give minimum tasks for the choice.

3) We have compared here topics which were learned only superficially with those which were well trained. In our situations, there were no differences in the factor structure except in the test 'Equations', and no clear differences as to retention. Learning processes have not been investigated, but understanding of mathematical principles seems to have its role in these processes. There were topics where training was of no help, especially to the low achievers.

4) We have not looked for a special programme for weak pupils, which would be suitable in remedial teaching, but there are clues as to reasons for the difficulties. The present writer has made an analysis of the mistakes in these algebraic performances in collaboration with Miss Meri Kaila, but this will be published separately.

There were many other results concerning the learning of special topics, which have not been published here. We can draw common didactical implications from these results only after learning processes are well known. In any case we have proceeded in such a way that didactical situations have been subject to manysided investigation by means of multivariate methods. So, necessary groundwork has been done for future didactical process analysis.

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Appendix A. COURSES OF ALGEBRA

In Grades 7 and 8 mimeographed sheets which were later published in a more developed form as an exercise book (Malinen, 1964) were used in Group 1 instead of a text book. This course included mostly elementary exercises. When using these sheets, the typical process of teaching happened in the following way: Firstly, there were repetition exercises which led to the new topic. Secondly, there were presented principles in the form of simple formulas. Thirdly, these new principles were exercised in simple situations. Fourthly, more advanced situations were presented later in a repetition phase. These phases are to be seen in many places also in this exercise book. New principles were often discovered by pupils in simple form and in most cases the verbal formalising came after the first exercise period. We cannot, however, call it a discovery method as described by Polya (1962) because there was no evidence about the common active discovery of pupils.

Pupils in Group 2 used during the experimental period the text book of Väisälä (1960). This book includes firstly a text where principles were defined or derived in a common algebraic form. In addition, some simple examples were presented. The whole text is presented in the first part and all exercises are in the second part of this book. The number of simple exercises is small and the teacher himself must present more. In this book there are more complex exercises and many of them could be studied only under the guidance of the teacher. A typical teaching process could not be seen in the book because text and exercises are presented separately.

In Grade 9 also pupils in Group 1 used the text book of Väisälä. Then, there was repetition of earlier tasks using more complex exercises. The repetition of functions was wider than in this text book. There was some theory of sets in Group 1 during Grade 7. This was used only a little during the later Grades in the solving of equations and in the theory of functions. The test 'Algebra XIV' controlled the learning of functions only in an elementary form. The more advanced learning of functions in Group 1 was not controlled. There was no control for instance concerning the theory of sets and calculating with quantities. These have not been taught in Group 2 and common tests would be impossible. Besides, our tests covered only partly the fields they were measuring.

As an example of the differences in the courses of algebra we present the way of teaching exponential expressions.

Group 1: Simple expressions like 2^3, $2^3 \cdot 2^2$ and their simplifying using the definition of exponent. After one month there were presented expressions like a^3 and x^4, and their values were defined by using special values for a and x. Yet later there was multiplication of exponential expressions in common formula like $a^m \cdot b^m = (ab)^m$ and $a^m \cdot a^m = a^{m+n}$. After these were exercised, corresponding formulas for division were derived.

Group 2: Firstly, the definition of exponent was presented: $a^m = aaa \ldots a$ (m times). Then, the laws of exponents were proved: $(ab)^m = a^m b^m$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ etc. After that these were exercised.

Another example of the differences in the courses is that the proofs presented in the test 'Algebraic Sentences' (Items Nos. 11–15) and many other corresponding proofs were treated in Group 2 already in Grade 7, but in Group 1 only in Grade 9.

The process of teaching should be equal in all classes as to the frames of teaching. It was not constant for all lessons but we can describe it as a teacher-centred method which includes the following phases:

1) Pupils present the results of their home exercises and the tasks will be discussed.

2) The teacher presents new principles using simple exercises (in Group 2 also theoretical considerations).

3) The teacher makes questions concerning the new principle and this task will be discussed.

4) All pupils do exercises which are presented on the blackboard or in the textbook. These will be discussed together.

The differences between Group 1 and Group 2 should be slight concerning the common organisation of lessons but there are differences because the exercises in Group 2 were more difficult than those in Group 1 during Grades 7 and 8. Then, more teacher-centred guidance was needed in Group 2. In Group 1 the working could be organised during short periods more individually, especially in Grade 7. Then, some pupils performed also additional topics concerning the number system and the theory of sets. This enrichment material is included in the experimental course material.

We have collected the written exercises of some pupils in each class. Then, these exercises have been grouped according to the same system as in the achievement tests. The number of written exercises varied a little between the parallel classes of the same group and still less between the pupils in the same class. We will omit this variance. We cannot measure the processing by using the number of these written exercises but we can evaluate the differences between the stimulus material of these groups. There have been difficulties in grouping exercises when there has not been a corresponding test in our testing programme. Because of the inaccuracy in grouping we have presented the numbers of exercises to the nearest 5.

Group 1, Grade 7

The order in the following list of written exercises is the same as the order in the teaching. Thus, it is possible also to indicate when the tests were presented. The exercises before the test are also in practice made before the test is presented. These test presentations are indicated in the text which is on file in the Pedagogical Library of Alppilan yhteislyseo and in the Institute of Education, University of Helsinki. The numbers of the exercises are given separately for each class so that we can see the differences between classes.

	Class	
	A	B
Presenting positive and negative numbers	20	15
Addition and subtraction of positive and negative numbers	140	100
Test Algebra I (September)		
Multiplication and division of positive and negative numbers	55	45
Test Algebra II (October)		
Combined exercises	35	30
Test Algebra III (October)		
Elements of the theory of sets	10	10
Symbols and common number marking in algebra	30	30
Calculating numerical values of expressions	30	25
Test Algebra IV (October)		
Principles of equation solving	15	15
Principles of exponential expressions	45	50
Principles concerning the combination of signs and parenthesis	25	25
Associative, commutative and distributive laws of algebra	30	30
Applications of the laws of algebra: addition, subtraction and multiplication	125	110
Test Algebra V (November)		
Addition and subtraction of polynomials	60	60
Test Algebra VI (December)		
Multiplication of polynomials (also exponential expressions)	95	90
Test Algebra VII (January)		
Division of polynomials, simplifying of simple rational expressions	115	110
Test Algebra VIII (April)		
More exponential expressions	30	40
Test Algebra IX (May)		
Repetition concerning the simplification of rational expressions	30	60
Test Algebra X (May)		
Total	890	845

Group 2, Grade 7

The numbers of the written exercises are in the following list. We have used here partly the same names for the groups of exercises though the content of exercises is not wholly identical in these two experimental groups. The tests were not well adapted to the teaching of this group. Thus, the exercises during Spring are mostly presented combined according to the content of tests.

	Class	
	C	D
Common number marking in algebra	40	45
Addition and subtraction of positive and negative numbers	145	110
Test Algebra I (October)		
Multiplication and division of positive and negative numbers	85	60
Tests Algebra II and III (October)		
Calculation of numerical values of expressions	30	25
Test Algebra IV (October)		
Associative, commutative and distributive laws of algebra	25	15
Test Algebra V (November)		
Simplifying of simple rational expressions	70	70
Test Algebra VIII (November)		
Principles of proportions	15	10
Principles of exponential expressions	80	65
Laws of exponents and their application	180	140
Test Algebra IX (January)		
Principles of equation solving	20	20
Solving of equations	45	70
Test Algebra XI (March)		
During Spring there were altogether the following numbers of exercises:		
Rational expressions (as in tests Algebra I and III)	25	10
Calculation of numerical values of expressions (as in test Algebra IV)	10	5
Application of associative, commutative and distributive laws of algebra (Algebra V)	25	10
Addition and subtraction of polynomials (Algebra VI)	20	—
Multiplication of polynomials (Algebra VII)	20	—
Rational algebraic expressions (Algebra X)	30	40
Exponential expressions (Algebra VII, IX)	35	50
Equations (Algebra XI)	15	20
Principles concerning formulas	25	—
Total	940	765

Group 1, Grade 8

Only a few new tests were presented during this school year. Thus, exercises have been collected into greater groups, but the exercises before each test were done before this test was presented.

	Class	
	A	B
Repetition exercises (Algebra III, IV, V, IX)	35	55
Principles of equation solving	25	30
Solving of equations	75	75
Test Algebra XI (October)		
Applications of equation solving: number problems and other verbal problems	50	35
Principles concerning calculating with quantities	35	35
Ratio and proportion	45	60
Variables and functions: elementary presentation (Algebra XIV)	15	5
Using the coordinate system (Algebra XIV)	35	30
Multiplication and division of polynomials (repetition as in tests Algebra VI and VII)	25	50
Total	340	375

Group 2, Grade 8

The corresponding list of written exercises in Group 2:

	Class	
	C	D
Repetition exercises (Algebra IV, VI, VII)	20	35
Addition and subtraction of polynomials	55	50
Test Algebra VI (October)		
Multiplication of polynomials (Algebra VII)	135	115
Division of polynomials	35	55
Solving of equations	40	60

Test Algebra XI (May)		
Applications of equation solving: number problems and other verbal problems	45	30
Total	330	345

Group 1 and 2, Grade 9

Pupils in both groups now had the same text book. There were many repetition exercises in Group 1 using this book. Thus, there were many more exercises in this group than in Group 2. Group 2 took more time to work with difficult problems. The tests were not presented at the same time in both groups, as can be seen in Table 6.7. The numbers of the written exercises are in the following list:

	Class			
	A	B	C	D
Repetition: polynomials, exponential expressions (Algebra III–VII, IX)	80	140	10	20
Repetition: rational expressions (Algebra VIII and X)	50	60	40	45
Factoring of polynomials	100	75	105	80
Rational expressions (Algebra XII)	55	40	45	55
Tests Algebra I–X, XII (January, February)				
Repetition: simple equations	20	5	25	15
Test Algebra XI (February)				
Complex equations	25	40	25	15
Test Algebra XIII (March, April)				
Systems of linear equations with two variables	15	20	25	15
Verbal problems	25	20	45	30
Test Problems III (April)				
The coordinate system and functions	25	30	15	10
Test Algebra XIV (May)				
Total	395	430	335	285

Appendix B. TABLES

Table 3.5. Correlation Matrix, Ability and Achievement Tests, Grade 7

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	38													
3	30	46												
4	17	10	21											
5	07	−00	11	69										
6	35	18	36	16	12									
7	16	13	31	06	07	36								
8	27	38	44	20	21	28	16							
9	41	46	53	30	25	38	31	43						
10	47	37	36	36	31	26	23	36	46					
11	40	43	40	37	27	25	23	46	53	74				
12	34	22	32	23	21	30	20	23	46	44	47			
13	04	20	24	16	11	04	07	15	38	30	33	19		
14	23	42	41	13	07	32	41	25	49	43	45	28	34	
15	37	37	41	22	27	33	27	38	58	70	72	52	34	51
	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Table 3.6. Centroid Factor Matrix, Ability and Achievement Tests, Grade 7

Variable No.	Factors I′	II′	III′	IV′	h^2
1	53	−12	26	−42	47
2	55	−27	−02	−01	37
3	62	−20	25	09	49
4	44	68	16	03	68
5	37	73	16	06	70
6	46	−13	36	−15	38
7	39	−19	34	10	31
8	54	−04	11	−05	31
9	75	−08	17	10	59
10	77	10	−29	−14	72
11	81	06	−29	−04	74
12	58	02	−05	−15	36
13	39	−01	−20	41	36
14	61	−24	05	28	51
15	80	−03	−24	01	70
Eigenvalues per variable	.35	.08	.04	.04	.51

Table 3.9. Centroid Factor Matrix, Common Ability and Achievement Tests, Grade 9

Variable No.	Factors I′	II′	III′	IV′	h^2
1	66	−15	−24	10	52
2	64	−14	−28	−07	51
3	63	−24	−05	12	47
4	44	59	−21	01	59
5	35	58	−28	−00	54
6	45	−15	−39	−18	41
7	53	−22	−08	23	39
8	49	−20	−23	−38	47
9	53	35	02	08	41
10	72	−02	25	−09	59
11	73	03	24	03	59
12	62	−02	09	24	45
13	61	13	42	−27	64
14	72	−15	−09	11	56
15	74	00	37	−03	68
Eigenvalues per variable	.36	.07	.06	.03	.52

Decimal points have been omitted.

Table 3.8. Correlation Matrix, Common Ability and Achievement Tests, Grade 9

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	52													
3	51	50												
4	25	30	18											
5	19	18	10	57										
6	36	40	28	21	18									
7	38	34	32	10	12	30								
8	37	44	28	11	13	44	24							
9	33	30	20	43	36	10	21	18						
10	39	37	45	27	17	27	36	34	35					
11	41	38	47	34	19	25	33	28	34	65				
12	41	36	37	26	13	28	37	17	36	42	51			
13	27	30	26	25	14	13	20	27	39	51	51	34		
14	54	50	51	18	23	32	49	38	40	47	49	42	36	
15	38	33	50	21	21	16	39	31	39	62	58	49	65	48
	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Table 3.11. Correlation Matrix, All Ability and Achievement Tests, Grade 9

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
2	52																					
3	51	50																				
4	25	30	18																			
5	26	36	35	69																		
6	17	17	07	59	48																	
7	51	60	51	09	20	07																
8	37	41	30	21	14	15	26															
9	45	37	33	10	09	14	25	31														
10	37	44	28	11	17	09	38	44	26													
11	31	38	36	18	24	17	45	25	15	33												
12	33	30	20	43	36	43	32	10	20	18	31											
13	39	37	45	27	29	18	43	27	34	34	36	35										
14	45	49	56	24	31	18	52	26	36	36	48	36	67									
15	41	36	37	26	36	20	40	30	36	17	39	36	42	54								
16	30	24	17	25	11	19	17	29	15	25	18	14	52	27	20							
17	26	31	28	13	26	17	39	10	26	32	27	40	47	50	36	22						
18	34	32	39	26	27	33	34	22	29	24	30	49	61	49	43	32	43					
19	32	29	36	19	28	19	37	16	25	24	38	35	58	63	43	28	53	60				
20	54	50	51	18	23	27	48	31	50	38	30	40	47	53	42	21	48	35	34			
21	30	24	35	05	11	13	36	17	23	33	29	34	47	50	35	26	46	48	55	37		
22	22	27	39	−02	14	05	35	10	13	07	38	04	20	43	29	05	28	20	29	31	21	
23	43	38	46	16	32	26	45	26	34	34	36	38	63	65	50	38	56	64	74	50	59	36
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

Table 3.12. Centroid Factor Matrix, All Ability and Achievement Tests, Grade 9

Variable No.	Factors I′	II′	III′	IV′	V′	h^2
1	63	−04	−32	09	11	53
2	64	−00	−42	−03	−05	60
3	64	−14	−23	−17	−12	52
4	39	75	−09	−00	−10	73
5	48	59	−07	−25	−12	66
6	36	62	06	03	12	53
7	65	−23	−17	−17	−02	55
8	42	−01	−36	30	−13	41
9	48	−11	−22	19	35	45
10	49	−12	−24	24	−08	38
11	54	−06	−08	−21	−21	40
12	54	32	13	−04	26	48
13	74	−05	20	21	−15	66
14	79	−14	07	−14	−08	68
15	63	03	00	−13	06	42
16	42	06	07	45	−30	48
17	61	−09	25	−06	16	47
18	68	07	32	12	05	58
19	70	−10	43	−04	−07	69
20	68	−09	−20	02	28	59
21	60	−21	30	08	06	50
22	40	−24	−03	−40	−12	40
23	80	−12	30	03	−02	74
Eigenvalues per variable	.35	.07	.06	.04	.02	.54

Table 3.18. Centroid Factor Matrix, Development Analysis

No.	Factors I′	II′	III′	IV′	V′	VI′	VII′	h^2_{12}
1	44	02	20	−43	08	−12	−30	530
2	62	16	13	−11	−01	00	−34	601
3	56	19	08	03	23	−14	04	569
4	61	26	−16	17	20	12	−09	652
5	38	−41	59	26	−06	10	−05	779
6	39	−29	57	36	−16	09	06	774
7	48	42	29	−29	01	34	12	733
8	44	44	38	−25	00	27	22	738
9	45	52	04	14	−38	−17	05	710
10	48	47	−10	17	−43	−10	−06	700
11	65	−02	29	03	16	−21	08	683
12	73	04	23	−04	18	−21	16	700
13	55	−29	18	13	−15	−07	−12	545
14	76	−21	−02	−14	−03	04	−23	748
15	74	−18	−01	−09	01	12	−08	700
16	73	−07	−11	09	−05	15	04	675
17	80	−28	−08	−07	−01	−03	−10	767
18	74	−10	−10	00	10	17	−05	736
19	75	−08	−08	09	−06	−00	14	673
20	55	−12	14	−33	−09	−23	17	573
21	64	01	02	−16	−14	−21	09	619
22	37	−14	−13	12	23	−11	18	413
23	61	−32	−17	−13	−20	00	18	664
24	68	27	−09	23	23	−08	03	687
25	69	27	−05	34	17	04	−08	720
26	70	26	−04	06	−04	−05	−19	628
27	78	02	−10	04	18	−02	04	720
28	84	−16	−23	−19	−01	04	04	843
29	81	−17	−31	05	−12	10	06	835
30	76	−12	−37	01	−14	14	09	815
Eigenvalues per variable	.41	.06	.05	.04	.03	.02	.02	.68

Table 3.17. Correlation Matrix, Development Analysis

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
2	48																												
3	30	36																											
4	07	51	49																										
5	17	22	21	04																									
6	07	25	19	18	73																								
7	35	35	36	33	16	12																							
8	26	36	25	28	15	21	71																						
9	16	29	31	31	06	11	36	37																					
10	10	38	27	32	−03	10	30	30	68																				
11	34	38	44	28	37	39	30	35	26	25																			
12	41	45	53	38	30	35	38	44	31	28	66																		
13	23	33	18	20	50	43	15	10	17	21	39	44																	
14	47	53	36	41	36	28	26	27	23	27	48	46	46																
15	39	47	30	39	34	28	35	24	24	27	48	49	41	62															
16	22	39	35	45	27	27	29	27	29	36	43	54	35	60	65														
17	40	44	40	40	38	28	25	18	23	29	52	54	53	74	67	61													
18	31	41	36	46	23	29	33	28	12	31	43	45	41	53	64	55	70												
19	24	41	43	47	25	34	27	25	30	33	50	55	34	52	59	65	62	62											
20	34	34	32	19	23	22	30	30	21	16	42	46	34	44	43	30	47	37	42										
21	34	41	31	37	15	26	28	29	34	37	40	51	36	43	47	42	50	51	51	57									
22	04	12	24	29	16	08	04	05	07	05	16	38	20	30	30	[illegible]	33	24	31	19	25								
23	24	27	24	26	22	25	16	13	18	20	34	41	39	52	46	51	60	39	51	44	34	25							
24	23	39	41	54	13	16	32	33	41	41	53	49	28	43	38	47	45	52	52	28	36	34	25						
25	17	45	43	60	21	22	35	31	45	44	46	53	36	43	48	53	46	53	50	21	33	29	22	67					
26	34	54	44	52	15	18	38	32	45	49	39	48	40	48	43	47	48	48	49	33	42	14	36	57	60				
27	33	38	44	48	24	18	35	32	31	31	56	58	42	57	59	51	63	58	55	37	42	34	49	64	61	58			
28	37	48	41	47	22	17	33	29	27	29	44	58	45	70	65	60	73	64	59	52	53	34	65	51	48	56	68		
29	22	40	36	47	26	19	26	19	31	38	41	45	49	65	65	62	69	64	65	43	49	35	61	50	56	52	67	82	
30	21	38	37	50	12	21	28	16	26	39	36	42	39	61	51	62	63	59	58	34	49	28	65	48	46	48	61	78	79
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29

Table 4.1. Correlation Matrix, Questionnaire 3, Grade 7

	1	2	2	3	5	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	04																							
3	09	32																						
4	−03	03	06																					
5	−04	44	11	28																				
6	08	−04	06	07	12																			
7	01	56	19	−07	45	−08																		
8	−05	15	25	11	40	13	23																	
9	08	70	37	15	43	02	39	15																
10	06	−13	−13	−03	−04	42	−11	20	−18															
11	07	44	11	09	39	−03	53	30	28	06														
12	07	62	24	05	45	−16	54	16	45	−21	38													
13	03	36	19	17	41	04	39	36	39	12	45	26												
14	07	56	39	20	41	24	37	12	67	−11	40	35	42											
15	−03	68	30	17	42	02	44	25	70	−07	35	46	42	64										
16	10	59	42	02	40	−07	42	23	60	−22	26	60	24	44	43									
17	15	16	03	07	17	44	04	15	21	24	11	02	22	33	16	05								
18	30	28	17	09	16	10	16	−01	33	−09	08	39	−06	29	13	51	−02							
19	03	55	21	06	44	20	66	36	46	05	61	49	59	58	54	38	33	17						
20	−09	66	18	−07	48	−11	53	23	54	−28	37	56	25	37	47	47	11	22	41					
21	−10	27	21	12	17	21	16	37	28	25	27	15	36	30	35	29	39	00	40	13				
22	10	62	33	09	44	01	57	24	68	−10	33	56	39	56	58	54	19	27	55	47	29			
23	08	55	16	04	54	02	49	25	52	−00	48	43	42	45	46	37	24	12	53	48	30	53		
24	−10	32	15	−08	27	−09	23	08	27	−35	−01	29	00	12	21	49	−15	32	10	38	−01	23	08	
25	06	53	56	15	30	−01	22	20	51	−13	23	39	22	47	40	51	18	25	28	41	17	35	26	26
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

Table 4.2. Centroid Factor Matrix, Questionnaire 3, Grade 7

Variable No.	Test Item	Factors I′	II′	III′	IV′	V′	h_5^2	h_8^2
1	(1) There are parts in Algebra which could be replaced by more practical topics	06	02	−23	42	−08	24	31
2	(2) Algebra is a difficult subject as compared with other subjects	82	−15	02	01	−13	71	73
3	(3) There could be more home exercises in Algebra	43	−10	−39	−27	14	43	58
4	(4) As compared with other subjects the syllabus makers have not succeeded in making Algebra interesting	14	14	−15	−18	17	10	32
5	(5) I do not like Algebra, for I am afraid of failing	63	09	14	−02	17	45	62
6	(6) I have really tried my best to succeed in Algebra	05	49	−37	19	11	43	49
7	(7) I am sometimes so depressed by my poor achievements in Algebra that I would like to cry	67	−04	40	16	04	64	66
8	(8) I am not at all satisfied with my achievements in Algebra in relation to my abilities in mathematics	37	30	07	−18	43	45	45
9	(9) Algebra is a dull subject as compared with other subjects	78	−11	−22	−08	−23	74	75
10	(10) I could try more in Algebra if needed	−13	60	−08	16	16	43	44
11	(11) Very often I do poorly in Algebra though I ought to do better	57	24	34	11	01	51	60
12	(12) I think the teaching in Algebra has progressed too fast	69	−27	13	17	08	60	61
13	(14) I am not at all satisfied with my achievement in Algebra in relation to my achievements in other subjects	55	38	18	−15	−01	50	55
14	(15) The studying of Algebra gives me a feeling of safety and is inspiring	72	14	−28	−03	−28	69	70
15	(16) As a rule I prefer to have other school work rather than Algebra	75	05	−03	−21	−23	65	69
16	(18) There are too great demands in Algebra at school	70	−32	−19	07	23	68	69
17	(20) I have really done my best to get my home exercises ready	25	50	−25	11	−06	39	47
18	(21) There are many unnecessary topics in Algebra	34	31	−34	43	14	54	57
19	(22) I am satisfied with my results in Algebra	74	32	18	13	01	70	71
20	(23) It is difficult to understand the principles in Algebra	68	−26	21	−02	04	57	70
21	(25) I am too lazy to study Algebra enough	40	42	−10	−14	16	39	43
22	(26) The studying of Algebra makes me irritable and restless	77	−03	−02	07	−11	60	65
23	(28) I need more exercises in elementary algebra	67	15	19	10	−09	52	53
24	(29) Only the talented in maths can understand the algebra syllabus as presented	33	−49	−04	−05	28	43	50
25	(30) There could be more difficult exercises in Algebra	57	−16	−34	−21	07	52	61
	Eigenvalues per variable	.32	.09	.05	.03	.03	.52	.57

Table 4.4. Correlation Matrix, Questionnaire 3, Grade 9

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2	12																							
3	−03	22																						
4	08	41	26																					
5	09	62	18	31																				
6	−01	01	−11	00	09																			
7	−02	44	15	13	39	−06																		
8	20	−02	−03	14	−02	17	−03																	
9	10	67	29	52	62	11	20	−03																
10	−04	−27	−11	−10	−19	44	−08	26	−18															
11	19	24	13	13	30	07	26	26	16	07														
12	08	52	10	33	55	13	29	−01	48	−08	35													
13	20	41	05	27	38	11	30	37	25	−13	42	26												
14	08	50	36	28	53	21	19	−00	65	00	29	38	23											
15	14	54	14	36	56	11	14	05	68	−07	12	41	33	47										
16	18	50	29	36	45	07	24	−00	58	−16	15	47	17	49	35									
17	−15	−10	−07	−01	−05	64	−02	17	02	34	13	08	04	11	−02	09								
18	30	39	11	29	14	03	17	08	34	−12	14	16	20	37	18	42	08							
19	10	44	25	26	45	24	40	23	37	00	48	47	52	44	34	36	29	23						
20	14	47	30	28	38	08	19	−02	50	−24	18	41	18	44	32	51	−04	32	29					
21	02	13	−04	09	26	50	17	25	26	14	27	21	18	20	26	29	52	13	34	30				
22	06	52	27	29	69	24	35	03	58	−08	27	56	33	51	49	60	08	21	51	46	35			
23	06	53	22	27	49	16	47	15	36	03	45	54	39	40	27	34	16	19	59	48	31	41		
24	10	41	13	28	34	−02	14	−07	45	−34	03	36	23	26	29	36	−14	07	21	32	11	41	17	
25	01	40	53	25	26	00	23	−04	37	−15	26	39	08	43	25	48	11	20	39	46	20	33	39	24
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

Table 4.5. Centroid Factor Matrix, Questionnaire 3, Grade 9

	Factors						
No.	I′	II′	III′	IV′	V′	h_5^2	h_8^2
1	17	−03	−20	18	38	25	33
2	77	−23	−09	09	−03	67	72
3	36	−21	11	−48	15	44	54
4	50	−12	03	10	26	34	47
5	75	−09	−07	20	−29	69	72
6	18	71	31	15	−04	66	66
7	44	−01	−30	−18	−27	39	46
8	10	39	−30	12	32	37	41
9	78	−17	27	20	07	75	77
10	−16	57	08	−01	04	35	46
11	42	27	−41	−16	05	44	49
12	67	01	−03	01	−20	50	51
13	48	14	−48	20	09	54	56
14	69	01	21	−07	11	54	65
15	63	−07	12	38	−00	56	63
16	69	−10	23	−07	14	56	61
17	11	74	27	−11	−01	64	68
18	40	−04	02	−00	44	36	49
19	67	29	−25	−14	−05	62	62
20	62	−13	14	−14	09	45	50
21	40	54	15	06	−04	47	53
22	76	03	11	10	−21	64	68
23	66	19	−25	−22	−13	60	61
24	46	−28	08	17	−09	33	45
25	55	−08	15	−47	06	56	58
Eigenvalues per variable	.29	.09	.05	.04	.03	.51	.57

Table 4.8. Correlation Matrix, Attitude Variables, Grade 7

	1	2	3	4	5	6	7
2	09						
3	26	35					
4	28	55	54				
5	20	20	16	37			
6	23	61	67	72	29		
7	51	15	50	33	18	39	
8	−00	−01	58	17	−12	34	48

Table 4.9. Centroid Factor Matrix, Attitude Variables, Grade 7

	Factors			
No.	I′	II′	III′	h^2
1	41	−06	60	53
2	57	45	−22	57
3	78	−26	−14	70
4	78	30	−02	70
5	34	32	26	28
6	84	15	−19	77
7	60	−36	31	59
8	44	63	26	66
Eigenvalues per variable	.39	.13	.09	.60

Table 4.11. Correlation Matrix, Common Attitude Variables, Grade 9

	1	2	3	4	5	6	7
2	08						
3	−01	46					
4	15	57	45				
5	18	23	11	38			
6	06	71	56	61	25		
7	52	08	02	21	20	11	
8	36	16	25	23	30	18	38

Table 4.12. Centroid Factor Matrix, Common Attitude Variables, Grade 9

No.	Factors I′	II′	III′	h^2
1	28	64	−09	50
2	76	−27	03	65
3	61	−26	−30	53
4	75	−07	17	60
5	42	20	38	36
6	80	−28	−04	72
7	32	63	−08	51
8	41	42	−10	35
Eigenvalues per variable	.33	.16	.04	.53

Table 4.14. Correlation Matrix, All Attitude Variables, Grade 9

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	16													
3	14	46												
4	22	56	45											
5	25	23	11	38										
6	18	72	59	60	25									
7	58	07	02	21	20	10								
8	−21	41	29	42	18	35	−27							
9	16	20	23	32	37	13	23	16						
10	42	27	27	27	16	21	36	−21	32					
11	21	30	37	42	25	29	25	13	30	58				
12	17	12	16	09	20	03	00	−01	20	23	03			
13	48	13	25	23	30	18	38	09	48	38	34	29		
14	38	54	51	57	26	46	21	37	47	44	43	16	63	
15	50	24	43	33	24	28	29	14	48	44	38	25	85	75

Table 4.15. Centroid Factor Matrix, All Attitude Variables, Grade 9

No.	Factors I′	II′	III′	IV′	V′	h^2
1	50	−46	−27	−18	−24	62
2	60	52	−21	−05	−09	68
3	59	32	01	26	−15	54
4	66	35	−14	−17	13	63
5	42	−01	01	−37	29	39
6	61	51	−24	−06	−21	74
7	38	−48	−40	−23	−04	59
8	28	56	30	−12	06	51
9	54	−13	23	−10	31	47
10	57	−30	−29	32	18	64
11	58	−02	−19	29	34	58
12	25	−13	16	−01	04	11
13	71	−44	36	−06	−11	84
14	84	07	18	06	08	75
15	79	−30	33	11	−19	88
Eigenvalues per variable	.33	.13	.06	.04	.04	.60

Table 4.17. Varimax-rotated Factor Matrix, All Attitude Variables, Grade 9

No.	Variable	Factors I	II	III	IV
1	Interest in School Work	01	*71*	07	09
2	Pleasantness of Algebra	*79*	04	09	17
3	Attitude to Algebra Syllabus	*56*	−11	16	38
4	Attitude to Success in Algebra	*71*	15	27	10
5	Dissatisfaction with Results	27	20	*49*	−08
6	Affective Attitude to Algebra	*83*	09	−00	13
7	Tendency to Try to Study Algebra	02	*70*	16	15
8	Capacity to Try More	*54*	−32	20	−20
9	Gain in School Success	14	08	*66*	15
10	Capacity in School Success	10	35	24	*67*
11	Dissatisfaction with Success	31	16	25	*57*
12	Marks Estimate	01	−06	*38*	17
13	Teacher Rating	11	33	*57*	21

Table 5.1. Correlation Matrix, Total Analysis, Grade 7

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	19																			
3	20	55																		
4	32	71	69																	
5	49	26	43	49																
6	08	19	31	24	31															
7	−12	06	32	15	17	38														
8	−07	05	34	19	14	30	46													
9	00	18	19	20	15	17	10	21												
10	16	22	20	24	23	07	−00	11	69											
11	08	11	29	21	27	35	18	36	16	12										
12	11	12	17	19	21	16	14	31	06	07	36									
13	−00	−20	28	23	28	27	38	44	20	21	28	17								
14	11	23	49	34	35	41	46	53	30	25	38	31	43							
15	21	41	53	57	50	47	38	36	36	31	26	23	36	46						
16	18	45	57	52	50	40	43	40	38	27	25	23	46	54	74					
17	26	19	40	36	34	34	22	32	23	21	30	21	23	46	44	47				
18	31	36	42	55	52	30	34	10	17	26	16	13	31	39	56	60	38			
19	14	18	38	28	41	17	41	43	21	13	35	45	32	53	43	46	21	45		
20	37	51	73	65	66	37	37	41	22	28	33	27	38	58	70	73	52	57	48	
21	40	21	43	43	80	36	24	33	23	28	30	26	28	51	57	56	41	49	43	77
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Table 5.2. Centroid Factor Matrix, Total Analysis, Grade 7

	Factors						
No.	I′	II′	III′	IV′	V′	VI′	h^2
1	33	44	−18	31	−07	−07	48
2	51	41	10	−47	−20	02	71
3	72	13	−05	−32	−12	−10	72
4	68	42	−02	−30	−14	−01	75
5	69	30	−23	39	07	07	82
6	50	−20	−06	−04	19	−31	50
7	46	−44	−10	−17	23	12	51
8	49	−55	−00	−02	−10	−06	58
9	39	−05	73	14	−01	−01	72
10	71	15	69	24	−07	03	71
11	43	−26	−06	10	−27	−19	41
12	36	−22	−14	13	−45	07	47
13	49	−29	07	−07	08	11	45
14	69	−32	02	01	−06	−04	61
15	79	03	07	−07	19	00	75
16	82	−03	07	−13	19	09	75
17	57	−03	−00	07	04	−34	53
18	67	19	−05	−02	21	22	65
19	59	−26	−11	09	−20	34	60
20	88	13	−12	−02	01	−03	85
21	76	12	−17	41	08	−01	83
Eigenvalues per variable	.36	.08	.06	.05	.03	.02	.64

Table 5.5. Centroid Factor Matrix, Total Analysis, Grade 9

	Factors						
No.	I′	II′	III′	IV′	V′	VI′	h^2
1	23	33	01	−18	24	−17	39
2	61	−06	41	40	05	01	73
3	63	02	37	16	00	−05	61
4	54	04	52	34	13	−13	75
5	58	57	−35	−11	16	−03	85
6	58	−43	−09	01	−01	06	56
7	60	−47	−09	07	02	−12	61
8	59	−23	−27	05	−10	−03	56
9	35	−24	35	−52	−08	−06	59
10	33	−09	34	−54	15	−16	61
11	37	−34	−20	−15	30	07	48
12	39	−13	−15	03	16	38	52
13	45	−30	−09	06	34	−08	50
14	52	−09	21	−29	−22	06	55
15	72	03	02	01	−02	17	64
16	60	−06	−17	−12	−24	15	52
17	67	24	19	06	−06	01	65
18	77	28	03	−02	−11	04	74
19	70	11	17	−07	−10	15	64
20	66	−22	−18	−03	08	09	60
21	61	−27	−27	17	−10	−19	64
22	53	−09	−20	−05	−13	−28	48
23	40	08	−32	09	−15	−28	42
24	85	29	06	10	−01	04	83
25	68	56	−25	−11	07	−00	88
Eigenvalues per variable	.34	0.8	.06	.04	.02	.02	.62

Table 5.4. Correlation Matrix, Total Analysis, Grade 9

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2	08																							
3	15	57																						
4	06	71	61																					
5	36	16	23	18																				
6	04	35	35	22	10																			
7	−02	41	33	30	14	52																		
8	08	27	25	16	25	51	50																	
9	08	19	23	17	−00	25	30	18																
10	22	15	27	25	12	19	18	10	57															
11	−04	08	14	08	18	36	40	28	21	18														
12	00	31	23	21	30	38	34	32	10	12	30													
13	10	27	27	21	14	37	44	28	11	13	44	24												
14	04	28	33	27	19	33	30	20	43	36	10	21	18											
15	17	45	40	36	39	39	37	45	27	17	27	36	34	35										
16	01	25	37	13	38	41	36	37	26	13	29	37	17	36	42									
17	19	46	53	44	45	27	30	26	25	15	13	20	27	39	51	34								
18	22	43	52	39	55	31	36	40	22	25	19	34	20	34	62	45	63							
19	26	46	47	39	37	34	32	39	26	27	21	31	24	49	61	43	53	59						
20	15	35	36	22	31	54	50	52	18	23	32	49	38	40	47	42	36	40	35					
21	05	36	29	27	29	51	60	51	09	04	26	27	38	32	43	40	30	34	34	48				
22	06	18	28	23	32	31	38	36	18	15	26	18	33	31	35	39	26	40	30	30	45			
23	12	19	17	09	36	22	27	38	−02	05	10	13	10	05	20	29	25	35	19	33	35	38		
24	27	60	59	54	62	38	33	50	21	21	16	39	31	39	62	49	65	75	64	48	41	41	32	
25	37	24	32	27	85	15	14	31	06	17	15	33	16	32	48	41	51	65	45	38	35	37	36	76
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

Table 6.2. Varimax-rotated Factor Matrix with 4 Factors, Total Analysis, Grade 9

No.	Variable	Factors I	II	III	IV	h^2
1	Interest in School Work	−06	*41*	05	14	19
2	Pleasantness of Algebra	26	06	*80*	04	71
3	Attitude to Success in Algebra	23	20	*65*	21	56
4	Affective Attitude to Algebra	10	08	*80*	10	67
5	Teacher Rating	18	*87*	04	−04	80
6	Number Series	*67*	−01	21	18	53
7	Syllogisms	*72*	−04	24	14	60
8	Arithmetic	*65*	20	13	02	48
9	Addition I, II	17	−00	11	*73*	57
10	Multiplication	07	11	10	*70*	52
11	Figures	*53*	02	−06	19	31
12	Form Boards	*47*	20	16	06	28
13	Cubes	*51*	02	18	08	30
14	Equations	27	19	24	*49*	41
15	Algebra I, II	*44*	39	39	19	53
16	Algebra V	*49*	34	12	18	40
17	Algebra VIII	20	46	*52*	16	55
18	Algebra X	31	*61*	42	17	67
19	Algebra XI	30	41	*44*	30	54
20	Problems I, II	*64*	23	18	15	52
21	Comparisons	*69*	16	21	−06	55
22	Operations	*48*	28	11	09	33
23	Understanding	*37*	34	04	−14	27
24	Mark in Algebra	35	*63*	54	11	83
25	Average Mark	21	*89*	16	03	85
	Eigenvalues per variable	.18	.14	.13	.07	.52

Table 6.5. Difference Matrix between Subgroups, Total Analysis, Grade 9

No.	Factors I	II	III	IV	Σd^2
1	−07	09	19	16	.08
2	07	−06	02	−14	.03
3	11	−02	03	−11	.03
4	−10	−06	−01	−12	.03
5	00	−02	−22	−23	.10
6	15	−06	−10	−05	.04
7	08	02	04	−18	.04
8	02	14	−12	30	.12
9	−02	29	−15	00	.11
10	00	43	−08	−10	.20
11	05	−21	09	−19	.09
12	−05	−04	−10	−11	.03
13	03	−16	28	−34	.22
14	24	21	−05	−04	.11
15	02	−13	25	−04	.09
16	06	04	−14	−04	.03
17	02	09	−13	14	.05
18	−04	13	02	13	.04
19	01	14	01	−09	.03
20	09	−14	−01	15	.05
21	07	11	01	32	.12
22	−08	19	22	00	.09
23	05	20	05	10	.06
24	09	−15	01	30	.12
25	02	−04	02	34	.12

Table 6.3. Factor Matrices for Subgroups, Total Analysis, Grade 9

	Group 1 ($n = 60$)				Group 2 ($n = 59$)				Total ($n = 119$)			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
1	−08	*48*	14	25	−01	*39*	−05	09	−05	*42*	05	15
2	32	04	*89*	−03	25	10	*87*	11	29	07	*88*	05
3	30	19	*73*	18	19	21	*70*	29	25	20	*71*	24
4	06	08	*86*	05	16	14	*87*	17	11	11	*87*	11
5	19	*90*	−04	06	19	*92*	18	−17	19	*91*	06	−05
6	*81*	−04	18	18	*66*	02	28	23	*74*	−02	23	21
7	*83*	−05	29	06	*75*	−07	25	24	*79*	−04	26	17
8	*72*	27	10	19	*70*	13	22	−11	*71*	20	15	04
9	18	13	06	*86*	20	−16	21	*86*	19	−01	13	*86*
10	07	35	08	*75*	07	−08	16	*85*	07	12	12	*81*
11	*58*	−08	−01	09	*53*	13	−10	28	*54*	05	−06	21
12	*46*	21	13	00	*51*	25	23	11	*48*	23	18	06
13	*56*	−05	34	−11	*53*	11	06	23	*54*	03	20	08
14	42	33	25	*54*	18	12	28	*58*	30	21	25	*55*
15	49	31	*55*	19	*47*	44	30	23	*48*	40	42	22
16	*56*	36	07	18	*50*	32	21	22	*53*	36	13	21
17	24	*51*	49	27	22	42	*62*	13	23	47	*54*	20
18	33	*67*	49	27	37	*54*	47	14	34	*62*	45	21
19	34	48	*49*	29	33	34	*48*	38	34	40	*48*	33
20	*74*	16	20	25	*65*	30	21	10	*70*	24	20	17
21	*79*	23	22	10	*72*	12	21	−22	*75*	17	22	−07
22	*47*	38	24	11	*55*	19	02	11	*51*	29	12	12
23	*42*	*42*	08	−11	*37*	22	03	−21	*40*	34	06	−14
24	42	*61*	*61*	29	33	*76*	60	−01	38	*68*	60	13
25	23	*92*	20	22	21	*96*	18	−12	22	*94*	18	03
Eigenvalues per variable	.23	.17	.16	.09	.19	.15	.16	.10	.21	.16	.16	.10

Table 6.4. Intercorrelations between Factors in Subgroups, Total Analysis, Grade 9

	Group 1 (N = 60)				Group 2 (N = 59)				Total (N = 119)			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
I	1				1				1			
II	.03	1			.04	1			.04	1		
III	.09	.02	1		.06	.09	1		.07	.05	1	
IV	.05	.19	.01	1	.06	−.18	.11	1	.06	−.01	.06	1

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